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Ludics, dialogue and inferentialism

Abstract: In this paper, we try to show that Ludics, a (pre-) logical framework invented by J-Y. Girard, enables us to rethink some of the relationships between Philosophy, Semantics and Pragmatics. In particular, Ludics helps to shed light on the nature of dialogue and to articulate features of Brandom's inferentialism.[1]

1. INTRODUCTION

A large part of Philosophy and notably of the Philosophy of Logic has ignored the recent revolution which has occured in logic over the last fifty years. Philosophers often think that such a revolution only concerns computer sciences and is merely technical, when in fact it brings to the fore essential features of language that a philosopher should not ignore. In particular, this revolution sheds a new light on the foundations of logical laws. This question repeatedly occurs in the philosophical debate. Sometimes relegated to psychology, that is to the organization of the brain, or to metaphysics under the aspect of transcendental norms, this question cannot be resolved until we have left behind the superficial analysis of logic, limited to the apparent laws of discourse, as we have received them from the first ancient logicians via the medieval tradition. Remaining tethered to this tradition leads

us to ignore the studies made within logic and calculus. By encompassing these studies, not only do we gain a deeper understanding of logic but also of language. In particular we may grasp the fundamental nature of language–*interaction*. About eighty years of research (at least from Gentzen, 1934) have lead to the huge importance of the concepts of *proofs*, seen as mathematical objects, and of *normalization* of proofs, also known as *cut-elimination* in the Sequent Calculus presentation of logic. While the study of proofs and their transformations gave rise to a proof semantics, particularly in the intuitionistic tradition of Brouwer and Heyting, their study in connection with *programs* led to the exploration of their dynamic aspects, as they are involved in the “proofs as programs” paradigm, largely illustrated by the Curry-Howard isomorphism (Howard 1980). This isomorphism not only provides an equivalence between formulae and types, proofs and programs, but also between normalization and reduction, that is the execution of a program. Seen through the perspective of computer sciences, logic appears as a domain of processes in interaction. Properties such as the Curry-Howard isomorphism, the Church-Rosser confluence, and the cut elimination theorem have become central properties in logic, but these are not accounted for in model theoretic semantics. In the eighties, the French logician Jean-Yves Girard (1987; 1989; 1989; 1995) generalized this line of research, essentially done in the context of intuitionism, by signaling the crucial role of structural rules in logic, paving the way for linear logic and its more recent successors, such as Ludics and the Geometry of Interaction.

In this paper, we try to show how this paradigm of research brings new concepts and tools that enable us to reflect on language and the Philosophy of language. As we shall see, this concerns not only the study of empirical objects such as ordinary *dialogues*, and therefore a study of pragmatics, through taking a slightly different approach than Grice's, but also philosophical approaches such as Rational Pragmatism (Brandom 1994, 2000), that have seen much discussion in recent years. In the modern logical frameworks that we consider, *interaction* is a two-faceted concept, one referring to the reduction of proofs, and the other to game-theoretic aspects. For instance, in Ludics, the framework invented by Jean-Yves Girard (2001), the main objects are not properly *proofs* but what Girard calls *designs*, that is, generalizations of *proofs* and *counter-proofs*, which can also be seen as strategies in games. The important point is that these games are not governed by rules which would be *a priori* given: rules are flexible and come to the fore in practice, only respecting some general geometric properties. Nor are they governed by some gain function: the game ends when players reach a situation where one of them endorses the move made by the other. We therefore reach a notion of game that is very suitable for the incorporation of wittgensteinian intuitions, and in addition, of claims by philosophers like W. Sellars and R. Brandom who argue, *contra* Wittgenstein, that language has a *downtown*, which consists of *giving and asking for reasons*.

2. LOGIC AND PROCESSES

2.1. *The Curry-Howard Correspondence*

According to the Curry-Howard correspondence, proofs in an intuitionistic system are programs and according to the Church-Rosser theorem, programs so obtained execute in a deterministic way, that is, the reduction of λ-terms is confluent (the result is not modified when we change the reduction strategy). Extensions of this isomorphism to other systems lead to interesting results and new interesting calculi such as Parigot's $\lambda\mu$-calculus and Curien-Herbelin $\lambda\mu\tilde{\mu}$-calculus (Parigot 1992; Curien & Herbelin 2000). In those "classical" calculi, a particular strategy must be chosen (for instance, the so-called *call-by-value* and *call-by-name* strategies) in order to keep the property of determinism. If not, computations may diverge. If we interpret "computation" as *semantic evaluation* of a symbolic form (a linguistic expression), we get the analogy with *semantic ambiguity*, that is several readings for one sentence (as in the case of *everyone loves someone*). In all cases, these terms (λ, $\lambda\mu$, $\lambda\mu\tilde{\mu}$) are proof encodings and their reductions refer to normalizations of the proofs they encode. We are thus led to study *proofs* in more general (or more specific) systems than intuitionism, that is to study:

- how proofs normalize,
- how they interact with each other.

2.2. *Logical Language and Ordinary Language*

Many of these observations can be transposed into the Philosophy of language, considering that cut-elimination could be the main feature not only of a *logical* language, but also of *ordinary* language. We shall even try to argue that *language* **is** *cut-elimination*, thus diverging from the traditional claim that language is primarily attuned to (the description of) external reality. Such a view has, we argue, a strong philosophical impact in that it may change the emphasis we put on some aspects of language, thus reinforcing the viewpoint, taken by Rational Pragmatism, according to which *inference* should be privileged over *reference*. Of course, a proof-theoretic viewpoint, like the one sustained by P. Martin-Löf, A. Ranta or R. Cooper (1984; 1994; 2012), can already deal with the inferential view according to which: "[o]ne can pick out what is propositionally contentful as whatever can serve both as a premise and as a conclusion in *inference*" but, as pointed out by Brandom, this is not the entire point:

> "[We] typically think about inference solely in terms of the relation between premise and conclusion, that is as a *monological* relation between propositional contents. Discursive practice, the giving and asking for reasons, however, involves both inter*content* and inter*personal* relations [...] The claim is that the representational aspect of the propositional contents that play the inferential roles of premise and conclusion should be understood in terms of the social and *dialogical* dimension of communicating reasons, of assessing the significance of reasons offered by others".
> (R. Brandom, Articulating Reasons, p. 166)

Our aim is therefore to show how a formal theory based on:

- *proofs* (premises and conclusions)
- a possible *game interpretation* of inference processes
- *interaction* (another name for cut-elimination)

can provide a rigorous account of inferentialism.
In passing, we will show that we need to extend the notion of proof in

order to get a realistic approach. Sellars' "inferential game of making claims and *giving and asking for reasons*" will then be seen as an interaction between processes which generalize proofs (in that they may, for instance, be *infinite*).

2.3. From Intuitionistic to Linear logic

It is often said that, if it is possible to translate proofs into λ-terms in the intuitionistic framework, it is because of the *constructivism* of intuitionistic logic (IL). In IL, sequents can easily be seen as *functions*: they have n inputs (as many as formulae on the left hand side) and exactly one output (or none when we consider the absurd ($\perp$) as a constant of IL). The cut-rule amounts to a *composition* of functions. Of course, in this framework there is no particular mystery about logical rules like *modus ponens*: "$A, A \Rightarrow B \vdash B$" simply means the application of a function of type $A \rightarrow B$ to an argument of type A, the result of which is known to be obviously of type B. Nevertheless, Intuitionnistic logic has restricted power and we could ask whether there exist other systems endowed with such properties of constructivism. While exploring the denotational semantics of the System F that he had invented (a system of second order logic that is very useful for computational purposes since, for instance, it provides a foundation for the use of *type polymorphism*), J-Y. Girard discovered linear logic, based on the use of particular morphisms ("linear" ones) between coherent spaces (Girard et al. 1989). Expressed as a syntactic calculus, linear logic emerged as *symmetric* (like classical logic) and *constructive* (like intuitionistic logic). In linear logic, sequents are no longer functions but reversible flows of information (n inputs, m outputs). They are "reversible" by means of negation ($^\perp$). As an essential difference with IL, in linear logic, the negation is involutive.

2.4. Linear Logic and Interaction

As is well known, the removal of the two structural rules of weakening and contraction entails distinguishing two families of connectives: *multiplicatives* and *additives*. According to the introduction rules for these connectives, contexts are cumulative in multiplicatives, but shared in additives.

For instance, the right rule for the additive "*and*" is the following:

$$\frac{\Gamma \vdash A, \Delta \qquad \Gamma \vdash B, \Delta}{\Gamma \vdash A \& B, \Delta}$$

stating that, by means of a multiset of resources Γ, in the same context Δ, if some user has A as well as B, she does not have A *and* B, in the cumulative sense, but she has the (active) choice between the two resources. Whereas & (the additive *and*) can be interpreted as an active choice, $\oplus$, the additive *or*, can be interpreted as a passive choice, that is, the user must be prepared to receive either A or B, but without knowing which one in advance. It may be shown that the De Morgan laws are preserved, in such a way that we have:

$$(A \& B)^\perp \equiv A^\perp \oplus B^\perp$$

$$(A \oplus B)^\perp \equiv A^\perp \& B^\perp$$

Thus, it clearly appears that negation ($^\perp$) does not play the usual role of "negating" (which is merely a La Palisse Truth) but, more subtly, the role of a *change of viewpoint*. Along the same lines, the usual law concerning implication expressed as : $A \multimap B = A^\perp \wp B$ (the counterpart, in the multiplicative dimension, of $A \Rightarrow B = A^\perp \vee B$) must be read as the rule of an exchange: if you give me A, I will give you B (in the same exchange, this is why $\wp$ is also called a "*par*", for *parallelization*). Thus begins a "game" interpretation of linear logic which has been developed by A. Blass, S. Abramsky, R. Jagadeesan and others (1992; 1994). But in order to go further with this interpretation, we must introduce more concepts, such as polarization.

2.5. Polarization

A second important distinction between connectives has emerged from some of Girard's insights and J-M. Andreoli's significant result (1992) on the *focusing* property of proofs in linear logic: some are said to be positive and others are negative. For instance, when applying the right rule for $\otimes$, coming from the bottom of the proof, we know that we have to guess a splitting of the context: Γ, the multiset of resources for obtaining $A \otimes B$, must be split into Γ_1, used to give A, and Γ_2, used to

give B, with $\Gamma_1 \cap \Gamma_2 = \emptyset$. Such a splitting necessitates a choice, that is, a decision from the user. A similar observation may be made concerning $\oplus$ since, in this case, in order to prove $\Gamma \vdash A \oplus B, \Delta$, we have to choose between $\Gamma \vdash A, \Delta$ and $\Gamma \vdash B, \Delta$. On the other hand, no such choice has to be made for either $\wp$ or &: the rules introducing these connectives are perfectly *reversible*. In other words, $\otimes$ and $\oplus$ have *active* rules, while & and $\wp$ have *passive* ones.
Following a theorem presented by Andreoli, any proof can be put in a *focalized* form, that is, a form which respects the following conditions:

- as long as there are still negative formulae in the sequent to prove, choose one
- when there are no more negative formulae, choose a positive formula and *focalize* it, that is, make active all the positive sub-formulae until there are none left.

A proof then becomes an alternation of sequences of positive and negative steps. Moreover, it is possible to regroup every sequence of (+/-) steps into one step, that amounts to applying a (+/-) rule for a *synthetic* connective. Then, any proof is displayed as an *alternation of positive and negative steps*. The calculus which is obtained, called *hypersequentialized* may be summed up in the following rules:
Formulas :

$$F = O|1|P|(F^{\perp} \otimes \cdots \otimes F^{\perp}) \oplus \cdots \oplus (F^{\perp} \otimes \cdots \otimes F^{\perp})|$$

Rules :

- axioms : $\overline{P \vdash P, \Delta}$ $\qquad \overline{\vdash 1, \Delta}$ $\qquad \overline{O \vdash \Delta}$
- logical rules :

$$\frac{\vdash A_{11}, \ldots, A_{1n_1}, \Gamma \quad \ldots \quad \vdash A_{p1}, \ldots, A_{pn_p}, \Gamma}{(A^{\perp}{}_{11} \otimes \cdots \otimes A^{\perp}{}_{1n_1}) \oplus \cdots \oplus (A^{\perp}{}_{p1} \otimes \cdots \otimes A^{\perp}{}_{pn_p}) \vdash \Gamma}$$

$$\frac{A_{i1} \vdash \Gamma_1 \quad \ldots A_{in_i} \vdash \Gamma_p}{\vdash (A^{\perp}{}_{11} \otimes \cdots \otimes A^{\perp}{}_{1n_1}) \oplus \cdots \oplus (A^{\perp}{}_{p1} \otimes \cdots \otimes A^{\perp}{}_{pn_p}), \Gamma}$$

where $\cup \Gamma_k \subset \Gamma$ (the possibility of $\cup_k \Gamma_k$ strictly inside Γ, allows weakening) and for all $k, l \in \{1, \ldots p\}$, $\Gamma_k \cap \Gamma_l = \emptyset$.

- cut rule :

$$\frac{A \vdash B, \Delta \qquad B \vdash \Gamma}{A \vdash \Delta, \Gamma}$$

Let us mention that:

- all propositional variables P are *positive*
- formulae connected by $\otimes$ and $\oplus$ are *negative* (the focalisation process leads to *maximal* decompositions of positive formulae)
- the general pattern $(\ldots \otimes \ldots \otimes \ldots) \oplus (\ldots \otimes \ldots \otimes \ldots) \ldots \oplus (\ldots \otimes \ldots \otimes \ldots)$ is not a restriction because of distributivity : $(A \oplus B) \otimes C \equiv (A \otimes C) \oplus (B \otimes C)$
- concerning the particular case of a synthetic connective of arity 1, where $(A^{\perp}{}_{11} \otimes \cdots \otimes A^{\perp}{}_{1n_1}) \oplus \cdots \oplus (A^{\perp}{}_{p1} \otimes \cdots \otimes A^{\perp}{}_{pn_p})$ reduces to only one term, the connective is noted: "$\downarrow$". We have then the two rules:

$$\frac{\vdash A, \Gamma}{\downarrow A^{\perp} \vdash \Gamma} \qquad\qquad \frac{A \vdash \Gamma}{\vdash \downarrow A^{\perp}, \Gamma}$$

where A is positive. In this case, $A^{\perp}$ is negative and "$\downarrow$" changes the polarity of the formula.
It has a dual, $\uparrow$, so that : $\downarrow(A^{\perp}) = (\uparrow A)^{\perp}$.

2.6. A Gaming Aspect

An *action* consists of playing a rule:

- either positive, amounting to the selection of a disjunct, then displaying its components
- or negative, amounting to the display of as many branches as there are disjuncts, displaying components of each one (with no active choice)

This stems from Andreoli's theorem that every positive (resp. negative) action is followed by a negative (resp. positive) action until axioms are reached. Now, what happens if we connect two attempts to build a proof, one pertaining to one player aiming to prove $\vdash A, \Gamma$, the other pertaining to another player, aiming to prove $\vdash A^{\perp}, \Gamma$ (or $A \vdash \Gamma$) instead? Let us take a particular case, where Player A wants to *prove* $X = (A_{11}{}^{\perp} \otimes ... \otimes A_{1n_1}{}^{\perp}) \oplus ... \oplus (A_{p1}{}^{\perp} \otimes ... \otimes A_{pn_p}{}^{\perp})$, while Player B wants to *deny* it. The two attempts may be plugged by the cut-rule. Then, the *cut-elimination procedure reduces the cut*, that is the previous occurrence of the cut-rule is replaced by one or several other occurrences of a lower degree (that is implying less connectives). This is evident in the fact that debate continues. In this particular case, the cut is transmitted to the pair made of the premise that A chooses to prove X and the corresponding premise assumed by B to deny X. This amounts to saying that, by a negative rule, B *previewed* all the choices possibly made by A. Let us suppose that A chooses $\Gamma \vdash A_{j1}{}^{\perp} \otimes ... \otimes A_{jn_j}{}^{\perp}$, that is the j^{th} component of the $\oplus$ (where $1 \leq j \leq p$), the cut is between this formula and the corresponding $\Gamma \vdash A_{j1} \wp ... \wp A_{jn_j}$, or in other words there are n_j cuts between formulae belonging to the sequents $\Gamma_i \vdash A_{jk}{}^{\perp}$ ($1 \leq k \leq n_j$) on one side and the sequent $\Gamma \vdash A_{j1}, ..., A_{jn_j}$ on the other side. Then, roles are interchanged, because each A_{jk} is positive, it is B's turn to perform a positive action while A performs a negative action. Of course, because there cannot be a proof of X and a proof of $X^{\perp}$, if one of the sequents is provable, the corresponding attempt (or *paraproof*) ends up by an axiom, while the other ends up by a termination rule which is not a proof rule but what can be called a *paralogism*: the *daimon*, formulated as: $\frac{}{\vdash \Gamma}\dagger$.

2.7. Geometrisation of Proofs

While proofs in intuitionistic logic are encoded into λ-terms, in linear logic, they are encoded into *proof-nets*, which are one of the main innovations due to linear logic. Generally speaking, proof-nets are graphs enjoying a particular correctness criterion. Figure 1 illustrates such a proof-net. The idea behind proof-nets can be formulated as follows. When we display the proof of a sequent in sequent calculus, the derivation produces many redundancies. Contexts are duplicated as long as

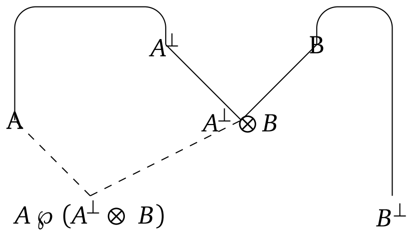


Figure 1: Proof net for $\vdash A \wp (A^{\perp} \otimes B), B^{\perp}$

formulae belonging to them are not active, moreover several derivations may be obtained differing only in a non-essential way (for instance non-relevant commutations of rules). The representation under the form of a net avoids this problem, thus providing the true essence of the proof. Having discovered that any sequential proof can be represented this way, it becomes possible to assert a new, geometrical, conception of proofs. Proof structures are graphs obtained by means of particular links associated with the connectives: the set of these links is the "bottom" of the proof attempt, or of its typing part. The graph is then completed by so called *axiom*-links, that is, edges linking atoms with opposite polarities (a and $a^{\perp}$ for instance): those links express the "proof" properly speaking. Cut-links are supplementary links which link two instances of the same formula, having opposite polarities, thus plugging two nets. Of course, not all proof-structures are proof-nets, that is, graphs that we can really associate with sequential proofs. A proof-structure that is not a proof-net is demonstrated in Figure 2.

If we compare figures 1 and 2, we see that in the latter the removal of a dashed line may suppress connectivity and maintain a *cycle* (see figure 3). while any such removal in the former graph keeps connectivity and eliminates cycles. From now on, we can use an easy geometrical criterion (known as the Danos-Régnier criterion (1989)) to discriminate proof-nets among proof structures:
A proof-net is a proof-structure enjoying the following criterion:

- *for every switching of a $\wp$ link, the graph is connex and acyclic*

(Switching is the selection of only one edge in a link). Cut-elimination

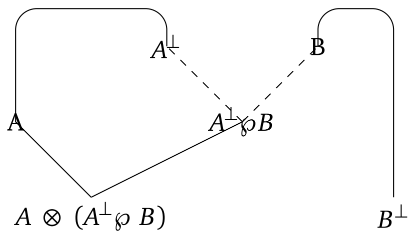


Figure 2: A proof structure which is not a proof net

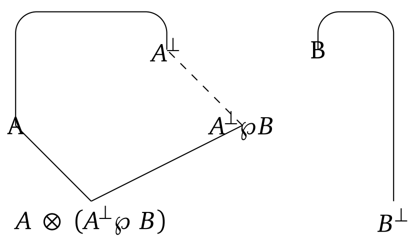


Figure 3: A cycle created by a switch

can be performed on proof-nets, as illustrated in figure 4. In stage (i), there is a cut between the two main formulae $A^{\perp} \otimes B$ and $A \wp B^{\perp}$. This cut is reduced in (ii) where it is replaced by two cuts, one between $A^{\perp}$ and A and the other between B and $B^{\perp}$. At this stage, the cut elimination algorithm faces the case of an axiom: the cut is removed in both instances, leading to (iii). All along the procedure, the correctness criterion has been preserved. Put in the reverse order, starting from a cut-free proof, a pair of dual formulae cannot be introduced, to avoid the creation of a cycle. This is a key point with regard to the foundations of logical laws, since, from now on, instead of starting from sequential proofs, then discovering the proof structures associated with them, we can consider the reverse: starting from proof structures which are proof-nets. In this way, the correctness criterion comes first and the derivations second. We can even say that rules are those particular kinds of morphisms which allow one to respect the correctness criterion, thereby producing a significant change of viewpoint in logic. Moreover, the origin of the criterion may be intuitively understood by means of the two fundamental results:

- acyclicity is an invariant of normalization
- normalization succeeds when every cut is eliminated

since normalization, for instance, would fail in the case illustrated in figure 5 (as the cut can never be eliminated). Because of the preservation of the criterion, the configuration in figure 5, if it is a result of normalization, necessarily indicates a cycle in a previous stage, therefore a wrong proof. Finally, execution explains correctness and at least partially, why rules are such as they are.
To sum up:

- proofs are alternations of positive and negative steps, but we may also consider other such alternations, such as *counter-proofs*, that is, attempts to deny a proof,
- geometrizing proofs leads to the pre-eminence of geometrical criteria (such as acyclicity), thus opening the field for more generalization of the concept of proof (*design*)

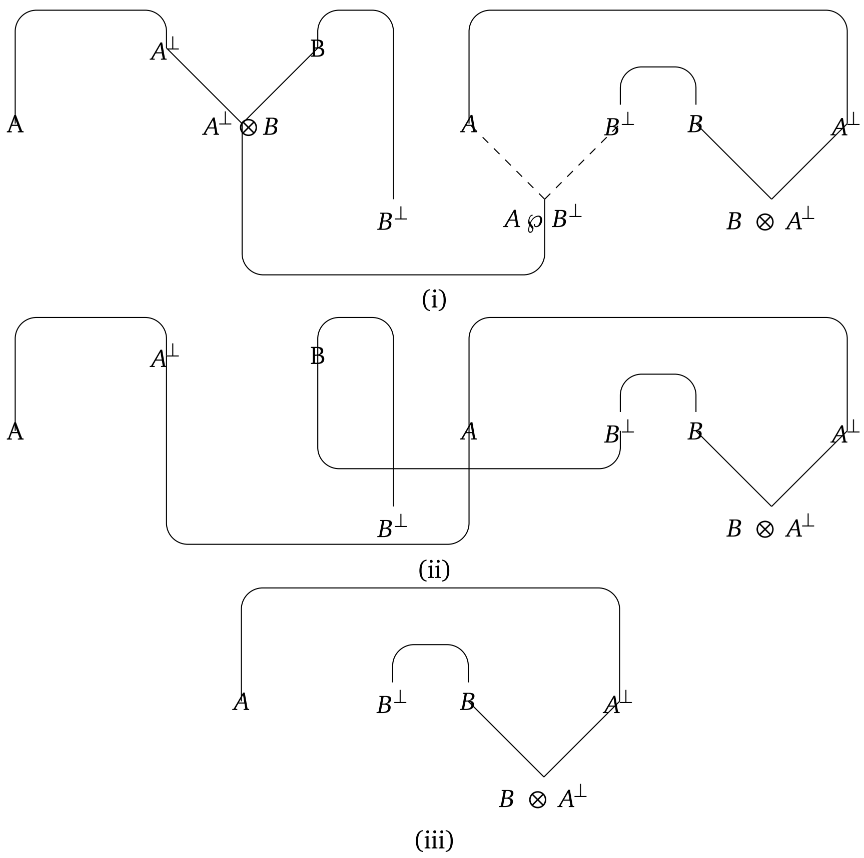


Figure 4: Cut-elimination in proof-nets

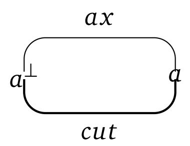


Figure 5: An elementary loop in cut-elimination

3. LUDICS

3.1. Rules and Designs

Ludics is an attempt to provide foundations for logical rules and connectives, starting from the properties we have seen in the previous section, which converge on the central idea of *interaction* (Girard 2001, 2006). In Ludics, we get rid of axioms and formulae, only keeping locations of formulae and sub-formulae, for geometrical purposes, notions like *actions* and their *polarities*, and of course the process of normalization. We can distinguish two rules (which may be seen as the skeleton of the two rules introduced in the hypersequential calculus):

- Positive rule

$$\frac{(\xi.i \vdash \Delta_i)_{i\in I}}{\vdash \Delta, \xi}\,(\xi, I)$$

where I is a finite set of integers (possibly empty) and the Δ_i's are pairwise disjoint and included in Δ.

- Negative rule

$$\frac{(\vdash (\xi.i_j)_{i_j\in I}, \Delta_I)_{I\in\mathfrak{N}}}{\xi \vdash \Delta}\,(\xi, \mathfrak{N})$$

where $\mathfrak{N}$ is a set (possibly empty or infinite) of finite sets of integers and the Δ_I's, not necessarily disjoint, are contained into Δ.

- and a third rule, Daïmon:

$$\frac{}{\vdash \Delta}\,\dagger$$

In those rules, *loci*, or addresses, are indicated by sequences of integers, sequents are limited to *forks*, that is, deduction relations with at most one locus on the left-hand side (all sequents could already be put in this form in the hypersequential calculus because of the involutive

$$\mathscr{F}ax_{\xi,\xi'} = \dfrac{\dots \quad \dfrac{\dots \quad \dfrac{\mathscr{F}ax_{\xi'_i,\xi_i}}{\xi'_i \vdash \xi_i} \quad \dots}{\vdash (\xi_i)_{i\in I}, \xi'}\,(+,\xi',I) \quad \dots}{\xi \vdash \xi'}\,(-,\wp_f(\mathbb{N}))$$

Figure 6: The Fax

negation and the fact that several formulae on the left-hand side could be considered linked by a ⊗, thus giving only one positive formula). *Negative* forks have a non-empty left-hand side. A *design* can be defined as a tree built only by using these rules and such that it always terminates (seen from the bottom) by the application of a positive rule. The fork at the root of a design is called its basis. A design is said to be positive (resp. negative) if its basis is positive (resp. negative). This is obviously a generalization of the notion of proof. The daïmon rule can be seen as a way to stop a design. Intuitively, this corresponds to an action of *giving up* (we shall see this in more details below). Such a generalization allows one to deal with infinite processes (e.g. an infinite design illustrated on figure 6).

3.2. Normalization of a Net of Designs

As evident from the three rules above, in Ludics, there is no explicit formulation of the cut-rule: a *cut* is therefore the mere coincidence of two loci of opposite polarities that share the same address. Normalization is then the process of cancellation of such pairs. Let us start from a net of designs the bases of which are, for instance:

$$\begin{array}{lcl}
\mathscr{D}_0 & : & \xi \vdash \alpha, \beta \\
\mathscr{D}_1 & : & \alpha \vdash \\
\mathscr{D}_2 & : & \beta \vdash \\
\mathscr{D}_3 & : & \vdash \xi
\end{array}$$

(where loci of the bases occur at most twice and the graph of the connections between loci has no cycle) then normalization may be per-

$$\dfrac{\dfrac{\dfrac{}{\vdash \xi.1.1}\,\dagger}{\xi.1\vdash}\,(-\{\{1\}\}) \quad \dfrac{\dfrac{}{\vdash \xi.2.1}\,\dagger}{\xi.2\vdash}\,(-\{\{1\}\})}{\vdash \xi}\,(+,\{1,2\}) \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\vdash}\,(+,\emptyset)}{\xi.2.1\vdash}\,(-,\{\emptyset\})}{\vdash \xi.2}\,(+,\xi.2,\{1\})}{\xi.1.1\vdash \xi.2}\,(-,\{\emptyset\})}{\vdash \xi.1,\xi.2}\,(+,\xi.1,\{1\})}{\xi\vdash}\,(-,\{\{1,2\}\})$$

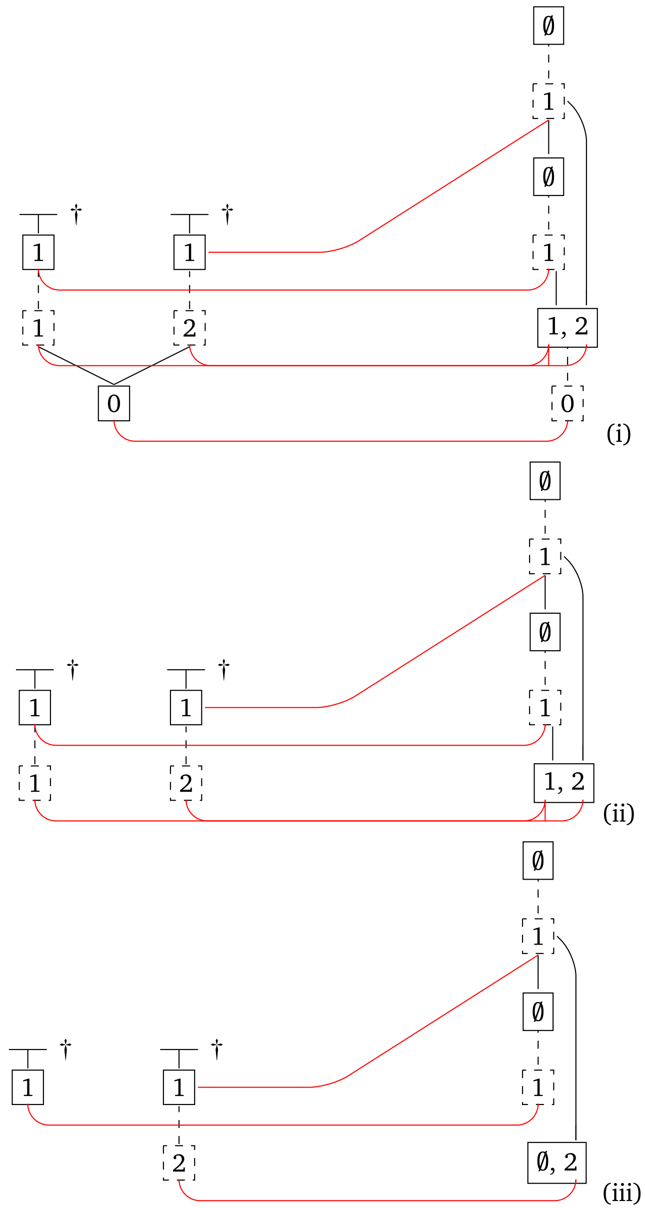
Figure 7: Two designs connected by a cut

formed. It may be explicated in the following way. Let κ be the last rule of the main design:

- if $\kappa = \dagger$ then normalization succeeds, the result is : $\dfrac{}{\vdash}\,\dagger$
- if $\kappa = (+,\xi,I)$ (in duality with $(-,\xi,\mathscr{N})$) :
  - if $I \notin \mathscr{N}$: normalization fails
  - if $I \in \mathscr{N}$: the process goes on.

An example is provided in figure 7, where two designs (one positive and one negative) create a net with a cut which can be reduced. For the sake of readability, we shall use a graphic representation of designs in figure 8. Positive rules are represented as embranchments in thin lines. They connect a positive fork, represented by a rectangle in a thin line, and negative forks represented by rectangles in a dashed line. Negative rules are represented as embranchments in dashed lines. Rectangles in dashed lines are indexed with their unique loci, while rectangles in thin lines are indexed by sequences of loci. If this sequence is empty, that means that there is no continuation above the marked locus. There can only be a continuation above a locus that is not yet marked (a new *focus*). In figure 8, both designs terminate on a positive action, as expected. The left design ends twice through the daïmon rule, and the right design ends through the positive rule labelled by the empty set.

Red thin lines connecting the two designs symbolize cuts: these are progressively removed, until reaching a situation (Fig. 9) containing only one last cut, which will disappear during the last step (not illustrated in the figure), in which the $\emptyset$ rule is confronted with the $\dagger$ rule. The result of this normalization is simply the empty fork $\frac{}{\vdash}\,\dagger$. In this situation, the two designs are said to be *orthogonal*.



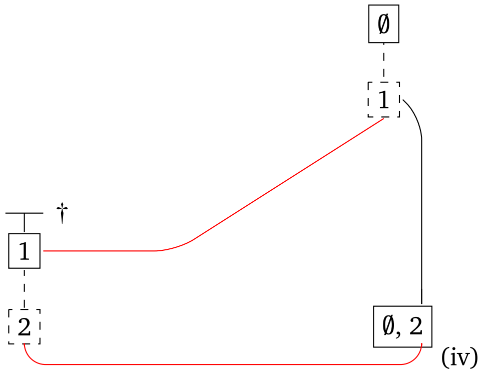
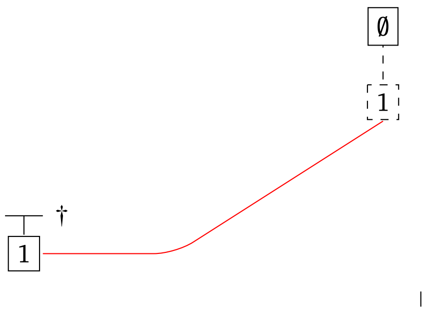


Figure 8: Normalization of a net of designs



(v)
then stop!

Figure 9: Normalization of a net of designs - Final steps

4. FROM LUDICS TO LANGUAGE

4.1. A Linguistic Interpretation of Normalization

We argue here that the normalization of the previous net of designs can be seen as an exact parallel to a situation of dialogue between two participants A and B, as illustrated below:

A - *let us speak of your father and mother*

B - *my father is dead...*

A - *... your mother?*

B - *she is retired.*

A - *I see...*

Actually, we can re-use the same graphical representation, with labels associated with utterances (illustrated in figure 10) progressing along the following lines.

(1) The first cut connects the question asked by A to its recording by B. Referring to figure 7, the locus of this question is ξ, and it opens two new loci ($\xi.1$ and $\xi.2$), *a priori*, one for the father, the other for the mother. The first step of normalization succeeds because B receives and accepts the question. It leads to a new situation, illustrated by (ii).

(2) Since A has created two negative loci (corresponding to two different ways of continuing the conversation) and B has recorded both, two cuts have to be reduced. The order in which they are reduced does not matter here. It happens that B first responds by speaking of her father (locus $\xi.1$) and that, at the same time, A, performing a negative action, has previewed the kind of answer that B is expected to give with regard to her father. The cut line connecting the two 1's (the one in the dashed rectangle and the one in the thin rectangle) can therefore be removed. This leads to the situation illustrated by (iii).

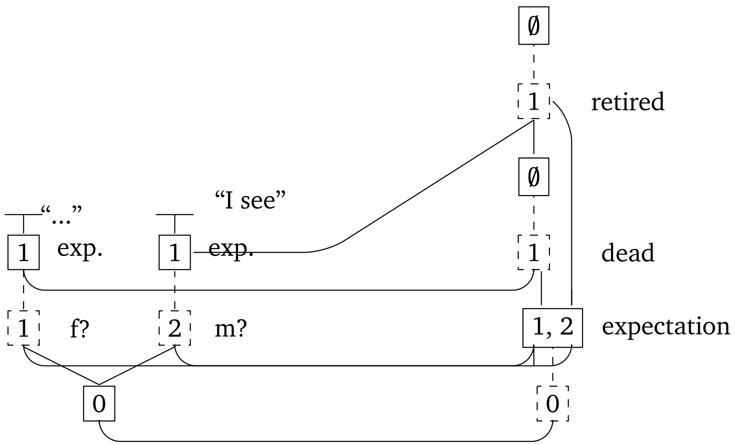


Figure 10: Dialogue as normalization

(3) The only positive action *A* can perform after the previous moves is to acknowledge *B*'s response by, for instance, repeating only the second part of her question (*... your mother?*). At the same time, *B* performs a negative move by which she stops any continuation above her father's locus (what she uttered is a mere *fact*). This combination of a fact and an acknowledgement results in a successful step altogether in the dialogue and the normalization. This leads to situation (iv).

(4) In situation (iv), the first focus (the father) has been given up, and it is time now to speak of *B*'s mother. Still, the response by *B* encounters *A*'s expectations, thus resulting in the removal of a new cut, leading to situation (v).

(5) In situation (v), *A* can only acknowledge the new fact provided by *B*, and the last cut is then removed, thus leading to a perfectly well-behaved dialogue.

(a) Emma: *We have a flat.*
(b) Robert: *Ah, I see*. (Pause) *Nice?* (Pause) *A flat. It's quite well established then, your . . . uh . . . affair?*
(c) Emma: *Yes*.
(d) Robert: *How long?*
(e) Emma: *Some time.*
(f) Robert: *But how long exactly?*
(g) Emma: *Five years.*
(h) Robert: *Five years?*

Figure 11: A sample dialogue, taken from Harold Pinter, *Betrayal*

4.2. Language as a Cut-elimination Process

Concerning dialogue, Ginzburg (2012) makes the following crucial observations, while discussing a very simple sample taken from H. Pinter's play *Betrayal* (see figure 11).

- "*Coherence*: each conversational move seems to cohere smoothly with its predecessor: questions are followed by answers which, in turn, raise new questions".

- "*Conciseness*: conversation is, by comparison with text, *a highly efficient medium*. Emma's affirmation of the well-established nature of the affair, Robert's wondering how long the affair has been going on, Emma's informing Robert that it has gone on for five years and Robert's astonishment at Emma's informing him this, all of this which takes 40 odd words of text to convey, takes a dozen words of dialogue."

- "*Radical Context Dependence*: Isolated from their occurrence in a dialogue many utterances lose most of their import. None of the utterances ((c)-(h)) could stand on their own in a text. Indeed, some utterances (e.g. ((d),(h)) resist a univocal sentential paraphrase. At the same time, in context, all these utterances seem readily comprehensible to the conversationalists."

It is true that all these features are amazing. Their prototypical example lies in the phenomenon of *non-sentential utterances* (from now on

Meg : *Is that you?*
Peter : **Yes**, *it's me*.
Meg : *What? Are you back?*
Peter **Yes**.
Meg : *I've got your cornflakes ready. Here's your cornflakes. Are they nice?*
Peter : **Very nice**.

Figure 12: Non-Sentential Utterances in a dialogue

NSU), that is, utterances frequently consisting of one word: *yes, no, perhaps* or even *bye, hmh...* and so on. Interpreting such utterances can only be done in context, but such a characterization is still too vague: these words explicitly make reference to the current interaction itself. As noted by Ginzburg, "the conventional meaning of a word or a construction involves notions that irreducibly involve reference to interaction - notions such as 'current issue under discussion', 'disengagement from conversation', 'acknowledgement of understanding' and 'ask intended reference of other's utterance'" (p. 5). A "grammar of dialogue", as proposed by Ginzburg, is, of course, a possibility. He proposes explicit conversational rules expressing the facts relative to the change of context due to a turn of speech, saying, for instance, that the pair (*speaker, addressee*), made of two *features*, is permuted at each turn of speech, or explicating what is the Latest Move to have been performed. However, we think that much of the information which would be contained in such a "grammar" is already present, without cost, in an interaction-based framework, where interaction is an *a priori* foundation, not constructed *a posteriori* by means of conventional rules. Let us demonstrate this with the extract from Pinter's *The Birthday Party* (figure 12). The dialogue from Meg's perspective can be

represented by the following design:

$$
\dfrac{\dfrac{\dfrac{\dots}{\vdash} \qquad \dfrac{\dfrac{\dfrac{..}{\vdash} \qquad \dfrac{\dfrac{\dfrac{\dots}{\vdash \Lambda} \quad \dfrac{\dots}{\vdash \sigma.0.1, \tau, \Gamma}}{\sigma.0 \vdash \Lambda}}{\vdash \xi.0.1.0.1, \sigma, \Lambda}\,c}{\xi.0.1.0 \vdash \Delta}}{\vdash \xi.0.1, \Delta}\,b}{\xi.0 \vdash \Delta}}{\vdash \xi, \Delta}\,a
$$

where a corresponds to the positive action "question": *is that you?*, b to *are you back?* and c to *are your cornflakes nice?*.
Peter's perspective is represented by the following.

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\vdash \xi.0.1.0.1.0}\,\emptyset}{\xi.0.1.0.1 \vdash}}{\vdash \xi.0.1.0}\,b'}{\xi.0.1 \vdash}}{\vdash \xi.0}\,a'}{\xi \vdash}
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{}{\vdash \sigma.0.1.0}\,\emptyset}{\sigma.0.1 \vdash}}{\vdash \sigma.0}\,c'}{\sigma \vdash}
\qquad etc.
$$

The net made of these designs normalizes just in case the first *yes* answers the first question, the second answers the second question and *very nice* is an option for "are your cornflakes nice?". Thus, as long as normalization is successful, we know that such replies can only be associated with the expectations expressed by negative steps in the other speaker's design, thus restricting the number of possible interpretations. It would also be possible to analyse the phenomena of *ellipsis* and *focus*, and of question / answer dynamics along the same lines.

5. FROM LUDICS TO INFERENTIALISM

5.1. Utterance and Game

It is interesting to compare Ludics (or its "spirit") with Brandom's inferentialism since both assign significance to the two notions of *proof* (or *inference*) and *game* (Brandom 1994, 2000). The latter is less explicit in Brandom's work, where it seems to be more metaphorical, as in the wittgensteinian notion of language game. Nevertheless, we argue that, even though metaphoric, Brandom's, as well as Wittgenstein's notions of game can be effectively recast in ludical terms. A similar position is noted by A. Pietarinen ((2007)) who remarks that "Girard's writings on Ludics carry Wittgensteinian undertones" (p. 273). Moreover, when seen from this inferentialist viewpoint, actions take on a different interpretation. Here, we move from the previous conception — according to which positive actions were conceived of as mere interventions in dialogue and negative actions as mere recordings — to a new conception, more precise and philosophically richer, where positive actions may be seen as *commitments* and negative actions as *entitlements*. In fact, performing a positive action in Ludics (distinct from the daïmon rule) is selecting a positive *locus*, as if selecting a token in a game, and from it, creating any number of negative *loci* (or none), as would be the case in a game that includes a rule explaining the circumstances under which you can move a token from a black square to a white square, to several, or to none. We then know that the negative loci (or the new tokens on negative squares) give rise to potential questions or objections from the other speaker, who may ask for reasons to say certain things, and that the speaker has to answer those questions and objections. Therefore, when performing a positive action (which is always followed by a negative action, unless it is the daïmon or a rule labelled by $\emptyset$), every speaker commits herself to providing reasons for what she says. On the other hand, by performing a negative action, every speaker limits the type of utterance the other speaker can choose for her to perform (e.g. either an answer or a new question). By doing so, every speaker gives entitlements to the other. In what follows, we shall consider a case in which the other speaker is fictitious and reacts as a "*scorekeeper*" in a game. In fact, his role is to give entitlements and, when acting positively, to react to every speaker's commitment either

by an acknowledgement (thereby increasing the speaker's score) or by a failure to acknowledge (which occurs when normalization fails). These notions of *score* and *scorekeeper* may be found in Brandom who says, for instance: "[u]nderstanding a speech act - grasping its discursive significance - is being able to attribute the right commitments in response. This is knowing how it changes the *score* of what the performer and the audience are committed and entitled to" (Brandom 2000, p. 165). Brandom is still more explicit when introducing the notion of scorekeeper:

> "Suppose we have a set of counters or markers such that producing or playing one has the social significance of making an assertional move in the game. We can call such counters "sentences". Then for any player at any time there must be a way of partitionning sentences into two classes, by distinguishing somehow those that he is disposed or otherwise prepared to assert (perhaps when suitably prompted). These counters, which are distinguished by bearing the player's mark, being on his list, or being kept in his box, constitute his score. By playing a new counter, making an assertion, one alters one's own score, and perhaps that of others." (Brandom 2000, p. 190)

This explanation can be correlated with the rules of Ludics. In Ludics games, counters are replaced by *loci*. We may consider that *selecting* one locus "has the social significance of making an assertional move". At any time, of course, any player has a way of partitionning loci into two classes: those he is allowed to select and those he is not allowed. This partitionning depends on the moves of the other player: when acting positively, if he wants *to keep convergence*, the first player has to select a locus in a range offered by the other one, and thus, if he succeeds, he improves his score (as well as the other's score — this shows the difference between Ludics and other game semantics. In Ludics, convergence gives points to both players so it is in their interest to cooperate).
We may also suggest that it is not only sentences that are counters, but also any part of speech. We may say that the utterance of *The swatch is red* is not simply submitting a proposition to evaluation by stating a "true" or "false" value, but *playing it as a token in a game*, and know-

ing that other players can ask for reasons for this statement, either by challenging the choice of the noun "swatch", or by contesting that "it is red". Only after this game has come to an end can the assertion be evaluated.

In this setting, *assertion* may be analyzed by assuming that *counters* are parts of speech and that when producing an utterance such as *The swatch is red*, the speaker is in dialogue with a score-keeper, who, at first, entitles the speaker to choose a THEME from a restricted range of topics allowed by the context. After the choice of a theme, the speaker expects it to be validated by the score-keeper, in a virtual positive move, followed by a new series of proposals concerning the PREDICATE, which is dependent on the selected THEME. The speaker can then select her own predicate from this series, waiting for a new validation, and perhaps other moves (involving assessment of *truth*, *modality* and so on).

5.2. Assessments of Truth and Normalization

Normalization may still be connected to the way rational pragmatism considers how assessments of truth work, as in the following passage from Brandom:

> Consider how assessments of truth work. Perhaps the central context in which such assessments classically arise is attributions of *knowledge* [...]
> In order for [a statement] to be *knowledge* that a scorekeeper takes another to have, that scorekeeper must adopt three sorts of practical attitude. *First*, the scorekeeper must attribute an inferentially articulated commitment [...]. *Second*, the scorekeeper must attribute a sort of inferential entitlement to that commitment [...]. What is it that then corresponds to the *third*, truth condition on knowledge? For the scorekeeper to take the attributed claim to be true is just for the scorekeeper to endorse that claim. That is, the third condition is that the scorekeeper himself untertake the same commitment attributed to the candidate knower. (Brandom 2000, p. 168)

We suggest that these three moves correspond to the three possible configurations in the normalization process. The first practical atti-

tude corresponds to the case where the "candidate knower" (here *A*) makes a positive move (distinct from the daïmon rule) while the score-keeper (here *B*) makes a negative one. In this case, the utterance is currently performed, *A* asserts something ("the swatch is red") while *B* records its content among her expectations. By doing so, *B* "attributes to *A* an inferentially articulated commitment" since *B* recognizes it as entering into the dialogical game. There is convergence if the representational[2] content of the assertion belongs to *B*'s set of expectations. *A*'s positive step may consist of a positive rule labelled by the empty set, provided that *B* acknowledges *A*'s statement as a *fact*. The second attitude corresponds to a second step, just above the previous one, that is, a negative step for *A*, which is opposed to *B*'s positive action. In this case, the utterance has been produced, and therefore the corresponding commitment has been made by *A*, and *B* can ask for reasons for each aspect under discussion. This amounts to *B* "attributing a sort of inferential entitlement to that commitment" since by challenging *A*, *B* attributes to *A* the power of drawing inferences in response to these challenges. There is convergence if *A* has previewed appropriate answers (*inferences A* is compelled to draw from her statement). And finally, there is the case where *A* performs a negative action and *B* performs the daïmon. *B* "acknowledges" *A*, or more precisely, he endorses the claim, or he "undertakes the same commitment attributed to the candidate knower". We thus have a deeper understanding of the role of the daïmon, and how Ludics, in particular, contrasts with other approaches such as classical game semantics. In traditional game semantics, the purpose of the players is to achieve a precise goal: the Proponent wants to show she is right when claiming that some thesis is true, and the Opponent wants to show the contrary. Those games are founded on an "objective" criterion which determines the winner (in Hintikka's games, the ultimate criterion is a model, by which it is possible to determine the truth or falsity of any elementary fact (Hintikka & Sandu 1997). In Lorenzen's games, it is a more internal criterion, relying on the fact that some player at some point has no rule to apply (Lorenzen 1960). Translated into extensive games, those criteria can be expressed as payoff functions. But, as Wittgenstein might say, not all games are games with a payoff function and a notion of a winner. Above all, most language games are not! At first glance, Ludics may also be seen as a search for the "winning" statement, particularly when viewed from the perspective of a proof system, such as the hypersequential calculus. After all, the purpose of such a system was also to get proofs, even if the notion of proof was analyzed from the perspective of interaction. But in the last sections we have seen that the object of a proof (that to which the concept of a proof can apply) can be significantly extended, so that it may now incorporate notions like *achievement, success* or *acknowledgement*. According to our final conclusion, inspired by Brandom's conception, a "proof" is an assertion, which is supposed to be checked by an interlocutor (a score-keeper) and the termination rule daïmon is used not "because there is no other rule to apply" but rather because the partner in the interaction consents *to endorse a claim*. That does not prevent one from referring to *truth* and *falsity*, to the extent that the interaction is taken to hold between a locutor and a (potentially) infinite number of participants, in this case, we can characterize a true statement as a statement for which all interactions end through the daïmon played by the dual participant. This clearly brings up similar statements by C. S. Peirce, whose principles are often compared with Wittgenstein's.[3]

5.3. *Incompatibility and Convergence*

Following this inferentialist direction, it is also worth noting that Ludics may go deeper than views expressed by Brandom (2008) concerning logical laws as grounded on an explicitation of discursive practices. According to Brandom's views, *incompatibility* plays the pre-eminent role. It is true that a commitment is such that the player who undertakes it is constrained by a logic of incompatibility (if she says that "it is red", she cannot say that "it is green" since these two judgements are incompatible). Nevertheless, it would be a regression to think of incompatibility in set-theoretical terms, thus assuming a non-contradiction principle at the source of logical laws when in fact we are looking for foundations for these laws. Moreover, as shown by D. Porello (2012), Brandom's concept of incompatibility does not take into account all the "standard" consequence relations we can imagine, but only the "classical" ones.[4] Such drawbacks may be avoided if incompatibility is defined in another way. Actually, as suggested by Porello, incompatibility must be defined on the basis of *symmetry between players*, according to which

each one has to agree on what counts as disagreement. This consideration calls for a formulation in terms of linear logic due to its particular interpretation of negation, as a change of viewpoint, so that involutivity ($A = A^{\perp\perp}$) means precisely that the propositional content of A requires agreement on which actions can challenge the content of A. Thus, the concept of interaction, and more precisely that of normalization (or *convergence* of the interaction) becomes the most appropriate for generating the consequence relations which emerge from the game of giving and asking for reasons.

Ludics contains important theorems which help us to distinguish between judgements, such as Girard's separation theorem according to which two designs $\mathcal{D}_1$ and $\mathcal{D}_2$ are said to be equal if and only if any counter-design orthogonal to one is orthogonal to the other. This provides us with a method to compare designs via their orthogonal sets : $\mathcal{D}$ is said to be "more defined" than $\mathcal{D}'$ if and only if $\mathcal{D}^{\perp} \subset \mathcal{D}'^{\perp}$, it is written $\mathcal{D} \preceq \mathcal{D}'$, and it can be shown that this relation is a partial order. Let us note that a *behaviour* is a set of designs which behave the same way with regards to the other designs. More precisely, if $\mathcal{D}$ is a design, $\mathcal{D}^{\perp}$ is the set of designs orthogonal to $\mathcal{D}$ (that is the set of all designs whose interaction with $\mathcal{D}$ converges), and $\mathcal{D}^{\perp\perp}$, which obviously contains $\mathcal{D}$, is a *behaviour*. In fact, we may see behaviours as *propositions* (see D. Porello for a similar view), and designs belonging to them as their possible *justifications* (or *reasons* if preferred). According to the separation theorem, a behaviour $\mathcal{D}^{\perp\perp}$ can be seen as the set of all the designs $\mathcal{D}'$ less defined than $\mathcal{D}$, and therefore a proposition as the partially ordered set of its justifications. If we consider two positive behaviours $\mathfrak{B}$ and $\mathfrak{B}'$ and $\vdash B, \vdash B'$ their respective bases, if $\mathfrak{B} \subset \mathfrak{B}'$, then each attempt to prove $\vdash B$ is also an attempt to prove $\vdash B'$ and if $\vdash B$ succeeds, so does $\vdash B$'. Thus, we obtain a natural notion of entailment, closed to *material* entailment. Of course if $\vdash B$ succeeds, $\vdash B^{\perp}$ does not, but attempts to prove $\vdash B^{\perp}$ are in $\mathfrak{B}^{\perp}$. Viewed from this perspective, we can say that *two dual behaviours express incompatibility*. There are actually two notions of incompatibility, strong and weak. Strong incompatibility occurs between two statements when the interaction of their designs *does not* converge. This can be illustrated with two statements which do not share their presuppositions ("is your sister a teacher?" - "But I have no sister!"). Strong incompatibility may be fixed by changing the designs. (For instance, in this example, we may unfold the first design by beginning with the question "do you have a sister?", if the other speaker answers "no", there is convergence). Weak incompatibility is in fact orthogonality.[5] We see then that in conformity with Brandom's aims, this "*incompatibility* semantics" yields consequence relations which make explicit our discursive practices, but as in Porello's model (based on phase semantics for linear logic), those relations are not limited to classical inference and are able to include more than just classical reasoning.

6. CONCLUSION

We have tried to show that Ludics is a promising framework which enables us to rethink many pragmatic, semantical and logical phenomena. We may summarize by saying that

- Ludics allows one to get rid of a conception of meaning based on model theory which takes truth, denotation and truth conditions for granted while many approaches to language (Peirce, Wittgenstein, Sellars, Brandom a.o.) are in favour of a more procedural way of grasping sentences' meanings.
- A consequence of this first direction resides in, in some sense, putting most of Semantics into the domain of Pragmatics : the use of words amounts to making actions, which are best depicted as *moves* in games.
- Another consequence is that Ludics allows one to account for inferentialism, overcoming some of the difficulties encountered by Brandom's theory and by proof-theory.

What is probably the most striking is that by delving deeply into the foundations of logic, the Ludics project has shed a new light on the foundations of language. Thus, this foundational exploration has given raise to concepts which can be dubbed "proto-logical" and are therefore applicable in the field of language as well as in the field of logic, thereby going towards a common foundation for both, something that we could ultimately name a foundation of $\lambda o \gamma o \sigma$.

Notes

[1]This work was completed with the support of the ANR Project LOCI number 0212 02.

[2]Brandom defines the representational content of a sentence, as opposed to the propositional content in these terms: "the representational aspect of the propositional content that play the inferential roles of premise and conclusion should be understood in terms of the social and dialogical dimension of communicating reasons, of assessing the significance of reasons offered by others." (Brandom 2000, p. 166.)

[3]For instance, his view that truth is a matter of long-term *convergence* of scientific research. cf. "Different minds may set out with the most antagonistic views, but the progress of investigation carries them by a force outside of themselves to one and the same conclusion. This activity of thought by which we are carried, not where we wish, but to a fore-ordained goal, is like the operation of destiny. No modification of the point of view taken, no selection of other facts for study, no natural bent of mind even, can enable a man to escape the predestinate opinion. This great hope is embodied in the conception of truth and reality. The opinion which is fated to be ultimately agreed to by all who investigate, is what we mean by the truth, and the object represented in this opinion is the real. That is the way I would explain reality" (Peirce 1982).

[4]For Brandom, a standard consequence relation is defined by two properties: *general transitivity* and *defeasibility*. The first may be expressed by the cut rule, and the second states that if a proposition B is not a consequence of A, then there is something which, when added to B but not to A yields an absurdity.

[5]Our notions of "weak" and "strong" incompatibility correspond to Girard's distinction between *refutation* and *recusation* with regard to the interpretation of negation.

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