9-1-2011

A Practical Method of Policy Analysis by Simulating Policy Options

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This article focuses on a method of policy analysis that has evolved from the previous articles in this issue.[1] The first section, "Toward a Theory of Educational Production," identifies concepts from science and achievement production to be incorporated into this policy analysis method. Building on Kuhn's (1970) discussion regarding paradigms, the second section, "Characteristics of an Achievement Production Theory and Model," describes a comprehensive, coherent, and unified theory and a mathematical model of achievement production substantially different from other theories and models. Using sample data, section three, "Example of the Policy Analysis Model," demonstrates the implementation of the model.

Toward a Theory of Educational Production

An Example of Scientific Method

To follow is a brief history of the scientific theory of gravity drawn from Feynman (1965, 17-20). In many ways, it parallels the motivation for and execution of the articles in this special issue. In addition, it highlights some fundamental differences in theory and models between the physical sciences and achievement production.

In ancient times, people believed that the planets circled the earth because earth "just had to be" the center of the universe. Later, Copernicus observed the planets moving in the sky and thought the planets, including earth, moved around the sun. The follow-up questions were: What pattern of motion do the planets follow—a circle or some other curve; and how fast do they move? Tycho Brahe thought he could help answer these questions by carefully recording how the planets move in the sky. From these data, alternative theories explaining the movement were developed. In essence, science was in transition from a philosophy to the collection and analysis of observations in order to develop better explanations.

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Kepler analyzed the observations made by Brahe and developed three propositions: The planet orbits are in the form of an ellipse; equal areas are swept in equal times; and the time it takes to go around the sun is based on a well-defined mathematical function. Meanwhile, Galileo, while testing the laws of inertia (rolling balls down an inclined plane), concluded that objects always move in a straight line unless some other force acts upon them. The force acting on the planets, Newton concluded, was gravity. The relationship is defined by his mathematical function: $F = G\ m_1 m_2 / r^2$.

As the ability to make accurate measurements increased, the tests of Newton's theory of gravity became more stringent. Indeed, the movement of the planets and moons could be accurately predicted by his mathematical function. Once the Newton law of gravity was confirmed through experiment, it was possible to build upon that knowledge to develop new knowledge. Based on the same mathematical function, Cavendish was able to determine the value of G, or "weighing the earth," through a laboratory experiment. Einstein later modified the Newton formulation when he discovered that energy and mass were related ($E = MC^2$); light would react to gravity and there is a "cosmic speed limit," the speed of light. The theory of gravity is tested every time an object is sent into space because the values within the equation change—there is a different set of initial conditions.

Still the theory of gravity is not complete. Physicists know that the laws on a small scale (the atomic level) are much different than the laws on a large scale (the universe). The analogy that the electron orbits the nucleus of the atom as the planets orbit the sun is incorrect. The Newton laws as modified by Einstein can predict with great accuracy the position and motion of the planets today and well into the future. On the other hand, there is no law predicting the position and motion of an electron in an atom. Quantum mechanics is built on what is called the "uncertainty principle"; that is, the position and motion of a particle cannot be accurately measured at the same time, but the probabilities can be measured with great accuracy. Today's sophisticated electronics are based on knowing these probabilities. A particle has even been named that controls all the movement in the universe—the Graviton—but to-date no one has been able to detect the particle and measure its properties. The endeavor to develop a complete theory of gravity is likely to be an endless journey.

There are several relevant points from the evolution of gravity theory:

- Over a long period of time, the thinking gradually shifted away from philosophy and beliefs to a science of observation, theory, and experiment. Once a theory was developed from observations, it was tested and verified by experiment. When the experiments more accurately predicted results, the old theories were replaced.
- A basic law can be expanded from the very simple situation to the very complex, e.g., the path of a thrown baseball to the motion of all the objects in the universe.
- The basic law demonstrates that all variables are not of equal influence. It is not necessary for every aspect of the complex system to be considered, only the most important. For example, an object with a small mass and a great distance from the earth (r^2) has virtually no influence of the orbit.

- With the basic law in hand, estimates of other variables within the system are possible. For example, Cavendish measured the coefficient of gravity, G in the formula, by suspending two balls from strings and measuring their attraction.
- With a strong theory behind the basic law, the theory gives direction to future research. In this way, the theories become more sophisticated over time.

The theory of gravity makes an interesting prediction. If the sun were to suddenly explode, reducing the mass, what would happen to the orbit of earth? Clearly the force would change, and the earth's orbit would change. There would also be other severe consequences. While the change of force would be automatic, the change would not be instantaneous. Rather, it would take about eight minutes—the speed of light—before earth would respond. By some magical and unknown process, "mother nature" knows exactly what to do. How does this scientific example apply to achievement production?

Shortcomings of Current Achievement Production Theory and Modeling

As seen from the gravity example, theories and mathematical models are representations of a phenomenon. Therefore, theories and models must be judged based on how well they characterize the phenomenon and how well they predict events, not based on a how well they reflect people's beliefs. Based on these criteria, there are some apparent shortcomings in the current achievement production theory and models.[2]

While each piece of class size research referenced in earlier articles in this issue has a research question, there is no fundamental theory being tested. What is implied is a "common-wisdom" theory: Reduced class size will *automatically* cause teachers to provide students with greater individual attention and, as a result, achievement will increase. This is not a testable theory. In order for a theory to be tested, it must be sufficiently concrete to allow observational data to be collected and analyzed. The individual attention theory is ill-defined, raising ambiguity regarding the actual theory being tested in class size research. What is implied by individual attention is a theory of changed behavior: By changing the class size or adding any type of instructional staff, staff behavior will automatically change, and so will the behavior of the students. As a result of these changes in behavior, achievement will improve. Before achieveme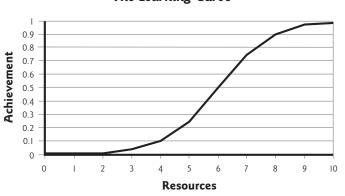nt can be expected to change, two critical steps must be taken; and neither step is included in the current theory or mathematical model. First, there must be a change in behavior by the instructional staff, and, second, there must be a change in the behavior of the pupils. The "automatic-individual-attention" theory and interpretation of the current achievement production model is not an accurate representation of the achievement-producing process. More likely, the theory involves a sequence such as a change of policies, a change of teacher and student behaviors, the practice of the new behaviors over time, and only then, a change in achievement.

There is another apparent shortcoming of current theory and modeling. According to learning theory and research, achievement does not change at a constant rate especially when there is an upper performance limit, i.e., a perfect score. There is a mathematical model representing the theory developed from observation and analysis: Achievement growth is proportional to the existing achievement level and to the difference between the existing level and the upper limit. (See Appendix B.) This model is in the form of a learning curve, illustrated in Figure 1. By assuming a constant rate of change, most achievement production research does not take the learning theory or the growth model into consideration. Indeed, there is no learning theory supporting a linear relationship between achievement and policy variables; there is only a statistical model with a linear feature. Most productivity research with the relationships proposed by Glass and Smith (1978),[3] i.e., increasing return to scale, and Hedges, Laine, and Greenwald (1994),[4] i.e., a constant return to scale, are inconsistent with this learning curve, and not an accurate representation of achievement growth.

Current achievement production research is mostly designed to test the hypothesis: Do resources (money or class size) make a difference? Studies are generally designed with one explanatory variable (expenditures or class size) and other control variables (e.g., socioeconomic status) and a statistical model to produce a kindly result. If the results are statistically significant, the policy implication is to "invest." In the cases of Glass and Hedges et al., they openly conclude that resources make a difference, and more resources make more of a difference.[5] Over a period of time, and partly due to these studies, a belief system was enhanced. Following this belief system, states and schools districts proceeded to make large investments in lowering class size.

Finally, current theories and models do not provide for the effective implementation of organizational or instructional policies. Because behavior does not change automatically, schools must rely on thoughtful policies as instruments of behavioral change. Since data are not collected regarding such policies, and little is known about their characteristics, these features are usually omitted from research efforts. There is evidence that organization behavior is consistently associated with academic performance and accounting for this behavior substantially increases the ability to predict achievement (Phelps 2009). Therefore, class size, organizational and instructional policies, and effective implementation of the policies all contribute to academic achievement. Theories and models not addressing the role of policies and behavior, the learning curve, or effectiveness do not fully characterize the complexity of achievement production. As a result, the models are less accurate in predicting achievement.

Figure 1
The Learning Curve

Characteristics of an Achievement Production Theory and Model

This section describes an achievement production theory and model with characteristics evolving from what are considered shortcomings of existing achievement theories and models. It also describes the steps for its implementation. Most importantly, achievement is a complex and dynamic system, which does not behave according to the physical laws determined by "mother nature." Just as a "gravity law" passed by Congress will not automatically change the behavior of the objects in the universe, the mere allocation of resources will not automatically result in improved achievement. While legislatures can allocate funds, they cannot change the shape of the "learning curve" or guarantee the effective use of the funds. In short, the achievement production model must be consistent with how schools teach and how students learn. It also must take into consideration the effective use of resources. This section is divided into three subsections: A policy-behavior-achievement (PBA) theory; the PBA model; and the PBA production model process, with steps for implementation.

A Policy-Behavior-Achievement (PBA) Theory

Because policy is the primary instrument influencing organizational behavior and behavior influences achievement, the proposed theory is: *Educational achievement is the product of all policies influencing staff, community, and student behavior and the effective implementation of those policies.*

There are several categories of policy variables, each with unique characteristics. Each of the categories influences some aspect of behavior.

- Resource or purchased variables include staffing quantity, staffing qualifications, instructional materials, and possibly special facilities.
- Family and community variables are represented by socioeconomic status (SES), which is divided into: the proxies used for measuring the association with achievement, but are beyond the control of schools, e.g., number of students receiving free and reduced-price meals, family income, and parent education; and the usually unmeasured behaviors which are also associated with achievement but are partially under the control of schools and community, such as motivation, discipline, and leisure reading.
- Process or effectiveness variables are organizational, per Levin (1997)[6], and instructional, per Walberg (1984).
- Incentive policy variables include extrinsic and intrinsic rewards for performance.[7]

The important role of behavior in achievement productivity is self-evident when looking at achievement at different organizational levels. Between school districts, there could well be differences in funding and class size accounting for the differences in achievement. Between school buildings within the same school district, the difference in funding and class size would most likely be less; thus the influence on achievement would be less. At the classroom level, there is no difference in funding or class size, but the achievement differences among students is still substantial. The different behaviors of the teacher, student, and family undoubtedly contribute to these achievement differences. This point is missing from other theories and models of achievement production. The contribution of behavior in response to policies is a key component of the policy-behavior-achievement paradigm.
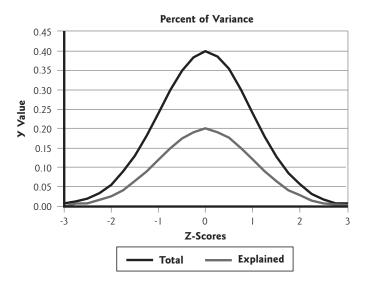
The family and community variable, SES, deserves special attention because of its potential role in influencing behavior. There is no fixed definition of SES. It is a concept for which proxy data are substituted, e.g., percent of students receiving free or reduced-price meals as a proxy for family income. Other proxies are common as well, e.g., parent or community education levels, student mobility, and attendance. In reality, these variables have no direct relationship with achievement. Instead, they are proxies for unobserved behaviors associated with achievement such as parent encouragement, time devoted to reading or homework, and rewards to do well in school. While the school cannot hope to change these proxy variables, it is possible through policy actions to influence the personal behaviors thought to be associated with achievement. This behavior aspect of the family and community variables is accommodated within the model.

It is possible to direct policies toward the educational staff, students, families, and in some cases, the community. In this context, a policy means a course of action to provide direction, assistance, supervision, evaluation, and rewards. An inventory of the various policies across the three groups of recipients will most likely reveal a disproportionate attention to what students should do. Less attention is paid to the instructional staff and little to families and the community, even though the benefits from such polices could be substantial. Because of attitudes regarding academic freedom to teach, or a reluctance to become involved in community and family affairs, a substantial potential may be missed.

Below is a succinct statement of the PBA theory:

- Achievement is the product of many behaviors: The student to study; the school staff to teach; and the family and community to provide a supporting environment.
- Behaviors are influenced by policies: What content the student studies and how they study; what content the school teaches and how the content is taught; and what contribution the family and community make to the educational process. (Learning does take place outside of the school setting.)
- The policies work in combination: Many complementary behaviors are required to produce or improve achievement.
- Some policies are more effective than others, and schools implementing more effective policies produce better academic performance.
- Effective policies can be different for various academic subjects and grade levels.
- Implementing some policies is more cost-efficient than others.
- In order to improve achievement, ineffective policies must be changed, and effective policy must be enhanced.
- Even effective policies eventually reach a point of diminishing returns.
- It is the responsibility of policymakers—school leadership, instructional staff, families, community—to select and implement the most cost-effective policies.

Figure 2
Total and Explained Variance in Student Achievement

Percent of Variance

[Figure: Graph with Y Value on vertical axis ranging from 0.00 to 0.45, and Z-Scores on horizontal axis ranging from -3 to 3. Two bell curves labeled "Total" (black, peaking at about 0.40) and "Explained" (gray, peaking at about 0.20).]

Legend: —— Total   —— Explained

The PBA Model

The policy analysis model builds on the principles previously presented in the theory. Importantly, it is not an analytical model, such as regression, designed to estimate the magnitude of relationships. It is a mathematical structure purposefully designed to represent the most important characteristics of school achievement derived from productivity research and from state school data. The purpose of the model is to accurately predict the largest achievement gains based on changes in the most cost-effective policies. In other words, the model is structured to optimize achievement by selecting the most cost-effective policies. This section addresses the following five issues: Representing effect size; measuring effectiveness; predicting actual achievement; the importance of initial conditions; and predicting a change in achievement.

Representing effect size. A critical element of the PBA model is the function representing effect size—the magnitude of the relationship between the policy variables and achievement. Because there is a built-in ceiling to achievement tests, the relationship between
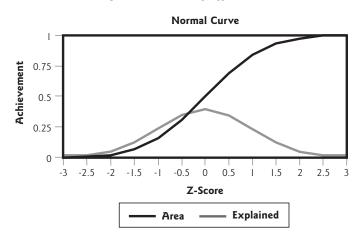
Figure 3
Representations of Effect Size

Normal Curve

[Figure: Graph with Achievement on vertical axis ranging from 0 to 1, and Z-Score on horizontal axis ranging from -3 to 3. An S-shaped curve labeled "Area" (black) and a bell curve labeled "Explained" (gray).]

Legend: —— Area   —— Explained

achievement and the variables is nonlinear. The percent of variance explained, the R^2 from a regression equation, is the logical function. It can be estimated by means of statistical analysis, and it allows for an optimization process not workable with linear relationships. The relationship between the total and explained variance is depicted by the following illustration. The achievement distribution (Total) and the distribution explained by the policy variables (Explained) are represented by normal curves, with explained portion being a proportion of the total.[8] (See Figure 2.)

The normal curve of the explanatory variable is mathematically integrated (summed to find the area under the curve). Thus, the explanatory variable is measured in standard scores (Z-scores), and achievement is measured in percentiles (area under the normal curve). The following illustration depicts the relationships between the distribution of the explanatory variable, the integral of the explanatory variable, and the achievement variable. For any value of R^2, the normal curve can be transformed to an S-shaped curve.[9] (See Figure 3.)

Measuring effectiveness. Previously, several categories of policy variables were listed, and each category has constituent variables. Because the constituent variables are most likely correlated, it is impossible to precisely measure the unique and common contribution each variable makes to achievement; that is, the contribution a classroom teacher makes to a student's achievement cannot be precisely separated from the contribution a special reading teacher or a teacher's aide might make to his or her achievement. Importantly, every constituent variable also has an effectiveness component; that is, not all administrators, teachers, reading teachers, or aides operate with equal effectiveness. Again, the constituent variables within the categories are usually correlated, so it is impossible to precisely measure the contribution effectiveness makes to achievement. Nevertheless, it is possible to estimate the total contribution effectiveness makes to achievement across all categories.

It is possible based on factor theory to measure the total achievement contribution—common and unique—of the conceptually and statistically related variables within categories, more appropriately called factors. The constituent variables for the Minnesota data were combined into factors: Staff quantity; staff qualifications; instructional materials; and SES. When achievement was predicted based on these factors, there was sizeable error, i.e., the difference between the predicted achievement and the actual achievement (the residual) was fairly large. Was the error systematic or random over time? In other words, did some schools consistently produce higher (or lower) achievement than what was predicted? The answer is yes, i.e., a portion of the error is systematic. Over a number of years, some schools consistently did something positive to produce higher than expected achievement taking into consideration the resource factors and SES. Some schools did the opposite, consistently producing lower achievement. This tendency to produce (or not to produce) achievement is measured by averaging the school residual over time (fixed effect estimation). This unobserved indicator of achievement production has been labeled "effectiveness" and most likely consists of some form of organizational and instructional behavior as proposed by Levin and Walberg.

Predicting actual achievement: The importance of effectiveness. The only way to accurately predict actual achievement is by comparing schools within the same state using the same achievement and explanatory variable measures. From these data, effect sizes for

Table 1
Estimates of Effect Size for SES, Resources, and Effectiveness

Achievement	SES	Resources	Effectiveness	Error	Sum
Mathematics	0.550	0.035	0.340	0.075	1.00
Reading	0.620	0.090	0.230	0.060	1.00
Mean	0.585	0.063	0.285	0.068	1.00

resource factors, SES, and effectiveness are estimated. The following production function predicts actual achievement (*AA*) from the resource factors and SES, as well as the contribution made by effectiveness, with a margin of error:

$$AA = \sum R^2 * \text{Resource factor} + R^2 * SES + R^2 * \text{Effectiveness} + \text{Error} \quad (1)$$

If effect size estimates (R^2) for the resources are used from other studies and they are higher than those from the state database, these estimates will predict achievement levels higher than the actual achievement. In this case, the production function can only be balanced to equal the actual achievement by reducing the contribution of effectiveness. In other words, if smaller classes are thought to make a larger difference and that difference is not reflected in the calculations for actual achievement scores, then schools must be ineffective in utilizing the full benefits of the smaller classes. This is a critical point worth restating. Lower class size predicts achievement only if the lower class size is implemented effectively. If a school does not meet the achievement level predicted by the class size, the only explanation is that they are ineffective. Conversely, if a school exceeds the achievement level predicted by the class size, they must be more effective in the implementation. Effectiveness is inextricably related to achievement production!

Regarding the theory of gravity, we know there is such as thing as a Graviton because we can measure its influence even though we do not know how it works. Regarding achievement productivity, we know there is such a thing as effectiveness because we can measure its influence even though we do not know exactly how it works. The following model explores this question: What are the possible characteristics of effectiveness, and how can they be incorporated into policy analysis?

The estimated effect sizes of the factors, taken from the Minnesota data set, are presented in Table 1. The staff quantity, staff qualifications, and instructional materials are included under the "Resources" factor. Because the factors are measured in terms of

the R^2, the sum of the factors must equal 1.00: If one factor is increased, another factor must be decreased. More importantly, if the effectiveness factor is not included, the error is increased.

When plotted, the effect sizes appear as S-shaped curves with the height of the curve proportional to the effect size. Effect size is analogous to a hill, the steeper the hill the larger the benefit. As the effect size gets smaller, it approaches a straight line. (See Figure 4.) As will be discussed later, it requires energy (resources) to "climb the hill."

Table 1 and Figure 4 highlight the critical differences between this PBA paradigm model and other models of achievement productivity. In this paradigm, the nonlinear effect sizes are bounded because of the inherent floor and ceiling in achievement testing. The position on the S-shaped curve determines the marginal effect size unique for each school rather than a constant effect size common for all schools. Also, the influence of a policy variable cannot be estimated without taking into consideration the effectiveness of implementation.

Importance of initial conditions. Returning to the theory of gravity and the work of Galileo, an object continues to move in the same direction and at the same speed unless another force is applied. The original direction and speed are called the initial conditions. By knowing the initial conditions and the speed and direction of the intervening force, the new direction and speed can be calculated. Applying this principle to achievement production, any model must first accurately determine actual achievement based on the initial conditions before it can forecast a change of achievement based on the change of those conditions.

The current standings of the resource and SES variables are considered the initial conditions. These initial conditions are determined by a school's placement within the total population, as measured by Z-scores and percentiles; that is, the contribution to achievement made by any variable depends where on the curve the school is
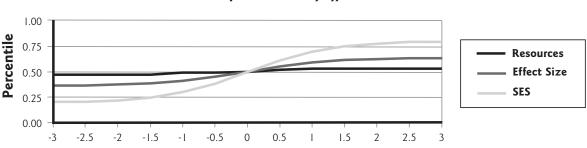
Figure 4
A Representation of Effect Size

situated because the slope is always changing. Identifying the initial conditions for the effectiveness variables is addressed later.

Predicting a change in achievement. After the model accurately predicts actual achievement, it must be modified to accommodate the changes of policy variables, which will predict later achievement. A critical element of the PBA model is the function relating achievement with the various policy options. Because a change of variables likely requires a change of funding, a cost variable is added to the equation:

$$PA = \sum R^2 \$ f_{(z)} \qquad (2)$$

where

PA = predicted achievement, and for every policy variable;

R^2 = estimated effect size;

$\$$ = incremental cost;

z = condition of the school on the policy variable;

and

$f_{(z)}$ is the nonlinear function representing the relationship between the policy variable and achievement.

A separate equation is constructed for each desired achievement outcome. The goal is to change various policy variables from their initial condition to their optimal condition to attain the highest potential gain in achievement, i.e., to change the value of Z. The change, or gain, in achievement is the difference between actual achievement (the old z) and predicted achievement (the new z).

Production Model Process: Steps for Implementation

The implementation of the model is divided into three broad steps: (1) Developing various policy options or scenarios, and simulating their influence on achievement, using estimates of effect sizes, estimated incremental costs, and the initial conditions of the policies; (2) evaluating the various scenarios based on the predicted achievement level; and (3) testing the success of the selected scenario through implementing the policy and measuring the accuracy of the prediction.

Developing policy options. The model evaluates achievement theories by simulating how various policies might impact achievement. Each combination of policy options is called a scenario.[10] The following resource and effectiveness factors with their constituent policy variables are available for inclusion in the simulation.[11]

- Resource variables—these variables, which are objects of funding, are identified in most state databases:
  o Staff quantity, e.g., ratios of teachers, aides, instructional support, and administrators to pupils;
  o Staff qualifications, e.g., education, experience, salary;[12]
  o Instructional materials.[13]
- Effectiveness variables: There is no direct identification or measure of process variables in state databases, but an indirect measure of an effectiveness factor is available for every school and is of a substantial magnitude. The following characteristics are assumed to be the components of the effectiveness factor and are called effectiveness variables in the remainder of the paper.
  o Instructional Effectiveness: Walberg identified these instructional characteristics—curriculum, method of instruction, instructional organization, home contribution, and time-on-task.

  o Operational Effectiveness: Levin identified these operational characteristics—measurable outcomes, incentives linked to outcomes, productive technology.

Evaluating scenarios. After possible policy scenarios are developed, they can be evaluated via simulation to estimate the predicted gain in achievement. Those portions of the policy scenarios judged to be workable based on predicted achievement gain, cost effectiveness, and practical operational considerations are refined while the impractical portions are dropped from further consideration. This refining process is continued until a final scenario is selected for implementation. The following example provides more detail regarding this process.

Testing Scenarios. This model is theoretical as it has not been tested in an actual situation. If persuasive, it gives direction as to how the model could be implemented and the results tested. First, more research into the characteristics of an effective of curriculum and instruction program would be valuable, as well as research into the characteristics of organizational effectiveness. Second, the model does not represent a solitary circumstance; rather, it is a template over which any circumstance or condition can be constructed. In essence, it is not the model that would be implemented and tested; it would be an individual scenario describing specific conditions that would be tested. Each scenario describes a set of school policies and makes an estimate as to the associated achievement. The selected scenario is tested by way of a case study where the implementation of the selected policies is monitored and the accuracy of the predicted achievement measured.

The case study approach would determine if the hypothesized characteristics of the policy options are actually present and influential. If they are, the scenario is directly confirmed, and the model is indirectly corroborated. As more evidence is collected, the model can be enhanced. To put it another way, the theories of Walberg (curriculum and instruction effectiveness) and Levin (organizational effectiveness), as well as those of STAR[14] and class size reduction experiments can be tested simultaneously within the same model. The model actually poses this research question: Can a specific level of academic achievement be accurately predicted by implementing a specific set of policies?

Example of the Policy Analysis Model

Prior articles in this issue center on the nature of the relationship between policy options and student achievement, and on estimating the effect size of the relationship. The previous section of this article described the theoretical bases and the specifics of the policy analysis model. The previous concepts and estimates are now transformed into a practical policy analysis model. Let there be no doubt, there are no magical answers. The suggested method demonstrates the difficulty in identifying the underlying data and assumptions required for any thoughtful policy analysis. It is often said that research is only as good as the data. In the case of policymaking, decisions must be made without the benefit of perfect data. Therefore, good policy depends on good judgments. These judgments are based on clear and comprehensive assumptions regarding the operations of the enterprise: What are the goals to be accomplished; what policies will influence behaviors; and what behaviors will achieve the established goals?

To follow is a description of how a policy analysis model might work in seven steps, as follows: Optimization principles; school

Table 2a
School Profile

| | Grade Level | | | | | | | Total | Cost ($) | |
	K	1	2	3	4	5	6		Average	Total
Student Enrollment	40	40	40	40	40	40	40	280		
Number of Teachers	2	2	2	2	2	2	2	14	$60,000	$840,000
Pupil/Teacher Ratio	20	20	20	20	20	20	20	20		
Number of Aides	1	0	0	0	0	0	0	1	$30,000	$30,000
Support (Reading Teacher)								1	$70,000	$70,000
Administrator								1	$70,000	$70,000
Total Instructional Staff								17	$80,000	$1,020,000

Table 2b
Statewide Statistics for Staffing Ratios

Staff per 1,000 Students		Mean	Standard Deviation	Z-Score	Percentile
Teachers	50.00	67.97	13.28	-1.35	8.80
Aides	3.57	22.14	20.51	-0.91	18.26
Support Positions	3.57	3.77	1.93	-0.10	45.90
Administrators	3.57	2.90	1.56	0.43	66.65

profile; estimating effect sizes; determining the initial conditions; the optimization process; interpretation of results; and the policy development process. The description of the process is followed by a discussion of the value of a policy analysis simulation and other considerations.

Optimization Principles

It is possible through mathematical programming to optimize the policy alternatives; that is, to select the best combination of policy alternatives based on their effect sizes, incremental costs, and initial conditions. For the optimization, a set of simultaneous equations is developed, one equation for each desired outcome including all of the influential policy variables. Another equation is constructed to calculate the cost of increasing the level of the policy variables. It is also possible for some variables to be decreased and the cost to be reduced. The goal is to select the optimal level for each policy variable that produces the highest level for the combined achievement outcomes while staying within an established cost limit.[15]

School Profile

To illustrate the PBA model, a hypothetical school is profiled. In reality, the data would be entered for the school in question along with the necessary statewide data. The information includes the number of students and staff in the various grades; average and total salaries; and the statewide means and standard deviation for the staffing ratios (staff per 1,000 pupils). From this data, the Z-scores and Percentiles (Ptile) are calculated. (See Tables 2a and 2b.) Additional data would be added to the profile if they were to be incorporated into the policy analysis. The school profile defines the specific initial conditions necessary to predict a change in achievement.[16]

Estimating Effect Sizes

The preceding article discussed the process of estimating effect sizes and provided estimates from various sources. The estimates from the Minnesota data set are the estimates used for the resource variables in this example. For the effectiveness variables, the estimates are those derived from Walberg. Because the Walberg estimates are not from the Minnesota data set, it is reasonable to substitute different estimates. Because there is an estimate for the effect size of the entire effectiveness factor, the average for the constituent variables could be a starting point, with adjustments made based on the judgments of the policymakers.

Determining the Initial Conditions

The initial conditions reflect the position of the school on the respective variables as measured first in Z-scores and then percentiles. The initial conditions of the variables must be set so the predicting equation equals the actual achievement. There are three groups of variables: Resource variables; effectiveness variables, including a portion of the SES variable thought to be subject to some policy influence; and fixed variables outside the influence of policy—the other portion of SES and error.[17]

The initial conditions for the resource variables and SES are standardized measures from the state database. The initial conditions of the effectiveness variables are unknown but can be estimated. First, the school must judge the "quality" level for each of the variables. Because there is no standardized measurement scale, one must be devised. To match the method of measuring resources, the starting point of the scale is a Z-score of 0, with a standard deviation of 1. Based on this scale, each effectiveness variable is rated either up or down. These quality values (Q) also meet another condition; when

Figure 5
*Setting Predicted Achievement to Actual Achievement by Adjusting the Initial
Conditions for the Effectiveness Variables*

	A	B	C	D	E	F	G	H	I	J	K		L
			INCREM		(R SQ)	(R SQ)		NOW			ACHIEVEMENT		
1	POLICY OPTION	ADD	COST	TOTAL	READ	MATH	TOTAL	RATIO	Z	PTILE	READ	MATH	TOTAL
3	RESOURCES												
4	Teachers	0.00	60,000	0	0.058	0.048	14	50.00	-1.35	8.80	-2.39	-1.98	-4.37
5	Aides	0.00	30,000	0	0.035	0.035	1	3.57	-0.91	18.26	-1.11	-1.11	-2.22
6	Support	0.00	70,000	0	0.010	0.010	1	3.57	-0.10	45.90	-0.04	-0.04	-0.08
7	Admin	0.00	80,000	0	0.010	0.010	1	3.57	0.43	66.65	0.17	0.17	0.33
8	EFFECTIVENESS						Q	C	Z	PTILE	READ	MATH	TOTAL
9	Curriculum	0.00	5,000	0	0.047	0.047	0.50	0.198	0.70	75.74	1.21	1.21	2.42
10	Instruction	0.00	5,000	0	0.083	0.083	0.00	0.198	0.20	57.85	0.65	0.65	1.30
11	Organization	0.00	5,000	0	0.003	0.003	-0.50	0.198	-0.30	38.14	-0.04	-0.04	-0.07
12	Home	0.00	5,000	0	0.070	0.070	0.00	0.198	0.20	57.85	0.55	0.55	1.10
13	Time	0.00	10,000	0	0.051	0.051	0.00	0.198	0.20	57.85	0.40	0.40	0.80
14	Change SES	0.00	15,000	0	0.050	0.050	0.00	0.198	0.20	57.85	0.39	0.39	0.79
15	FIXED							0.198					
16	FIXED SES		N/A	N/A	0.500	0.500	0.00		0.00	50.00	0.00	0.00	0.00
17	ERROR		N/A	N/A	0.083	0.093	0.00		0.00	50.00	0.00	0.00	0.00
18			TOTAL	$0					SUM		-0.21	0.21	
19	Students	280	TARGET	$100,000	GAIN	0.00%			PREDICT		49.59	50.41	100.00
20	Per Pupil			$3,643		$0			ACTUAL		50.00	50.00	100.00

combined with the values of resource variables, they must predict actual achievement. To accomplish this, a parameter (C) is introduced which adjusts all the effectiveness variables, assuring that the equation equals actual achievement. This method answers the question: What initial conditions for the resource and effectiveness variables will predict actual achievement?[18] The initial condition for the error term is set to 0.

Actual achievement (AA) equals the sum of the resource variables (R) plus the sum of the effectiveness variables (E) adjusted by parameter (C), and the error:[19]

$$AA = \sum R_{(z)}R^2 + \sum E_{(z\,=\,Q+C)}R^2 + Error_{(z\,=\,0)} \qquad (3)$$

If the effectiveness variables were judged artificially high, the predicted achievement would be higher than the actual achievement. In essence, the parameter C becomes a "truth detector" for the quality judgments, and makes the appropriate adjustment. Actual achievement can be high only when the both the resource and effectiveness variables are at high levels. (See Figure 3.)

For total predicted achievement to equal total actual achievement, the initial condition parameter for the effectiveness variables (C) is .198.[20] (See Figure 5, Column H, Lines 8-14.) If actual achievement were higher than 100, the effectiveness parameter would increase, i.e., the school operations are more effective, and vice versa.

Optimization Process

The next step is to identify the most cost-effective policy options by automatically determining the best option through an optimization process. Many spreadsheet programs have an optimization feature. In Microsoft Excel, it is referred to as the "Solver." By identifying the target as the maximum gain in achievement, Solver will determine the best allocation of funds among the policy variables based on effect sizes, incremental costs, initial conditions, and an overall spending constraint.

In mathematical programming, the parts of the model are called the object function and the constraints. The object function is a mathematical function representing the goal to be attained, in this case the sum of various achievement measures. There are two types of constraints. The first type includes the mathematical functions representing the relationship between the various explanatory variables and the various outcomes. The second type includes the boundaries—maximums or minimums—for the variables. Importantly, there must be a boundary or upper limit to at least one variable, in this case cost, or there can be no end or conclusion to the calculations. Solver requires these parameters:

- Set Target Cell To:
  - The cell contains the object function or value to be attained, in this example the sum of the achievement measures.
- Equal To:
  - Maximum, minimum, or value. In this example, maximum is marked. The purpose is to find the values producing the maximum predicted achievement.
- By Changing Cells:
  - The range of cells is the values of the policy variables to be changed in order to obtain the maximum achievement level.
- Subject to the Constraints:
  - The maximum-, minimum-, or equal-to-values that reflect the assumptions regarding the school operations. Most importantly, the value of the additional cost must not exceed the predetermined value or target value. In this example, the constraints are set to prohibit any reduction of existing staff or a reduction in any of other policy variables.

Figure 6
Optimization of Policies

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	POLICY OPTION		INCREM		(R SQ)	(R SQ)		NOW				NEW		
2		ADD	COST	TOTAL	READ	MATH	TOTAL	RATIO	Z	PTILE	TOTAL	RATIO	Z	PTILE
3	RESOURCES													
4	Teachers	0.00	60,000	0	0.058	0.048	14	50.00	-1.35	8.80	14.00	50.00	-1.35	8.80
5	Aides	0.00	30,000	0	0.035	0.035	1	3.57	-0.91	18.26	1.00	3.57	-0.91	18.26
6	Support	0.00	70,000	0	0.010	0.010	1	3.57	-0.10	45.90	1.00	3.57	-0.10	45.90
7	Admin	0.00	80,000	0	0.010	0.010	1	3.57	0.43	66.65	1.00	3.57	0.43	66.65
8	EFFECTIVENESS						Q	C						
9	Curriculum	2.06	5,000	10,284	0.047	0.047	0.50	0.198	0.70	75.74			2.56	99.47
10	Instruction	2.77	5,000	13,851	0.083	0.083	0.00	0.198	0.20	57.85			2.77	99.72
11	Organization	1.52	5,000	7,583	0.003	0.003	-0.50	0.198	-0.30	38.13			1.02	84.54
12	Home	2.71	5,000	13,541	0.070	0.070	0.00	0.198	0.20	57.85			2.71	99.66
13	Time	2.31	10,000	23,052	0.051	0.051	0.00	0.198	0.20	57.85			2.31	98.94
14	Change SES	2.11	15,000	31,689	0.050	0.050	0.00	0.198	0.20	57.85			2.11	98.27
15	FIXED							0.198						
16	Fixed SES		N/A	N/A	0.500	0.500			0.00	50.00				
17			TOTAL COST	$100,000										
18	Students	280	TARGET	$100,000	GAIN	9.80%								
19	Per Pupil			$3,643		$357								

Figure 6 illustrates the optimization process. Solver changes the values in the ADD cells in column B, producing the highest gain in achievement while simultaneously taking into consideration the cost. To explain the process, the simultaneous elements are by necessity described sequentially.

The change of conditions and costs constraints. The heart of the simulation is displayed under ADD of the spreadsheet, which determines the new conditions of the policy variables producing the maximum increase in predicted achievement. The starting point of all variables is zero; therefore, a zero under ADD indicates no change in condition. An increase of the policy condition incurs a cost. This cost, which appears by variable under TOTAL (column D), was calculated by multiplying the values under ADD by those under INCREM COST (Incremental Cost) in column C. These are summed to reach a TOTAL COST of $100,00 (column D, line 17). The TOTAL COST is limited to a user-determined value or TARGET cost. For this example, the TARGET cost has been set at a $100,000 increase (column D, line 18). PER PUPIL indicates that expenditures are $3,643 per pupil (column D, line 19). This represents a GAIN of $357 per pupil, or a 9.8% increase. Based on the new policy conditions, the NEW levels are provided (columns K-N):

- TOTAL refers to resource variables, which is the number of teachers, aides, support personnel, and administrators;
- RATIO is staff per 100 students;
- Z refers to Z-score;
- PTILE refers to percentile.

The new Z-scores and percentiles are also provided for the effectiveness variables (columns M-N, lines 8-14). Note that when the percentile rankings for some variables move to a point of diminishing returns (≥90%), the other variables become more cost effective.

In this example, actual achievement for reading and mathematics is set at the mean, or 50th percentile, with a total of 100. Because the optimization is yet to take place, there are no values for the change from the initial conditions (ADD) or increased costs attributed to changing the initial conditions (TOTAL).

The change in predicted achievement. In simple terms, the optimization identifies the most cost-effective policy variable and increases the policy value to a point of diminishing returns, at which point it moves to the next most cost-effective variable. It moves through this sequence until the funding target is reached. At that point, the total achievement gain is at the maximum level.

The information regarding the achievement levels before and after the optimization is provided in Figure 7. For each of the policy variables and for each subject area, reading (READ) and mathematics

Table 3
Verification of Effect Sizes in Simulation

Variables	R^2	
	Reading	Mathematics
Resource	0.113	0.103
Effectiveness	0.254	0.254
SES	0.550	0.550
Total	0.917	0.907
Error	0.083	0.093
Grand Total	1.000	1.000

Figure 7
Achievement Gains through Optimization

	A	B	C	D	E	F	G	H	I	J	K
1	ACHIEVEMENT	READ	READ	READ	MATH	MATH	MATH		TOTAL	INCREASED	GAIN/
2		BEFORE	AFTER	GAIN	BEFORE	AFTER	GAIN		GAIN	COST	$10,000
3	RESOURCES										
4	Teachers	-2.39	-2.39	0.00	-1.98	-1.98	0.00		0.00	0.00	0.00
5	Aides	-1.11	-1.11	0.00	-1.11	-1.11	0.00		0.00	0.00	0.00
6	Support	-0.04	-0.04	0.00	-0.04	-0.04	0.00		0.00	0.00	0.00
7	Admin	0.17	0.17	0.00	0.17	0.17	0.00		0.00	0.00	0.00
8	EFFECTIVENESS										
9	Curriculum	1.21	2.33	1.12	1.21	2.33	1.12		2.23	10,284	0.22
10	Instruction	0.65	4.13	3.48	0.65	4.13	3.48		6.95	13,851	0.50
11	Organization	-0.04	0.10	0.14	-0.04	0.10	0.14		0.28	7,583	0.04
12	Home	0.55	3.48	2.93	0.55	3.48	2.93		5.85	13,541	0.43
13	Time	0.40	2.50	2.10	0.40	2.50	2.10		4.19	23,052	0.18
14	Change SES	0.39	2.41	2.02	0.39	2.41	2.02		4.04	31,689	0.13
15	FIXED										
16	Fixed SES	0.00	N/A	N/A	0.00	N/A	N/A		N/A		
17	TOTALS	-0.21	11.57	11.77	0.20	11.98	11.77		23.55	100,000	1.50
18	Optimized		61.57			61.98		123.55			
19	Predicted	49.79			50.20			100.00			
20	Actual	50.00			50.00			100.00			

(MATH), the BEFORE and AFTER achievement results are expressed as percentiles. The achievement gains for reading and mathematics are provided under READ GAIN (column D) and MATH GAIN (column G) respectively. These are summed under TOTAL GAIN (column I).

Based on the assumptions in this example, the predicted achievement gains due to the effectiveness variables (curriculum, instruction, organization, home, time, change in SES) as seen under TOTAL GAIN are positive, ranging from 0.28 to 6.95 percentile points. However, no gains are shown for resource (staffing) variables. All of the effectiveness variables would have to be at the point of diminishing returns before the resource variables would become cost-effective. The increased cost for each variable is found in column J. To assist in the evaluation, column K provides the results of cost-benefit analysis, giving the gain in predicted achievement for each policy variable based on an investment of $10,000 (GAIN/$10,000).

Verifying effect size. There is a running tabulation of the R^2 entered into the optimization model. In order to protect against the tendency to overestimate the influence of the policy variables, the sum is provided. (See Table 3.) These should and do sum to 1.00, including the error. These effect sizes correspond to those of the Minnesota analysis. It is important to start with a state database in order to establish some reasonable ranges for the effect sizes. As was pointed out earlier, having good SES indicators is critical in obtaining good estimates for the other factors.

The constituent variables should fit within the limits of the resource and effectiveness factors listed in Table 1. Remember, .05 was moved from the SES factor to the effectiveness factor for the previously stated reasons. Even with the resource variable in the simulation being higher than the factor from the data set, the resource variables are not as cost-efficient as the effectiveness variables. It is clear that if the effectiveness factor were omitted from

the analysis, the error factor would be substantially larger and the predicted achievement much less accurate.

Interpretation of Results

If the results from this model are only as good as the assumptions, what are those assumptions? The PBA paradigm and model stand on two pillars: The relationship between achievement and the policy variables is nonlinear; and the most effective policy variables are those influencing a change in behavior. The degree of trust in the results from the PBA model is directly proportional to the commitment to these assumptions. Trust does not work in the reverse direction; that is, trust in the assumptions is not directly proportional to the commitment to the results. In other words, one must trust the results because the theory and model are persuasive rather than trust the theory and model because one likes the results. As the reader will soon see, the results from the PBA models are quite different than those from other models.

The critical parameters in the model are effect sizes, initial conditions, and incremental costs. Particular attention should be paid to the veracity of these parameters. The illustrative simulation identifies instruction as the best investment and the other effectiveness variables as the most cost-effective, but why? It is because the effect sizes for the effectiveness variables are larger than those for the resource variables, and the incremental costs are less. The estimates of the effect size for the effectiveness variables originated with Walberg and are supported by the analysis of the residuals, the fixed effects. The other element is the initial condition. The model assumes the initial condition for the effectiveness variables can be established by the judgments of policymakers. Just in case, they are adjusted by the effectiveness factor (C), so they are at least in the "ballpark." Clearly, this assumption must be tested.

The final element is the incremental costs. Could the incremental costs be wrong? Doubtful! While there is a certain amount of

guesswork in the other parameters, the incremental cost estimates should be far more accurate. There is an instructive "rule of thumb": If the incremental cost of one variable is double the incremental cost of another, then the effect size of the more costly variable must be double in order for the benefit of the two variables to be equal. In other words, incremental cost, the most accurate parameter, is the most influential. The model provides a potential gain per $1,000 calculation to show the relative potential of each variable. With the assumed initial costs, the effectiveness variables clearly have greater potential benefits. Remember, the potential benefits are tied to the initial conditions. If the initial conditions for the effectiveness variables are high, their potential benefits diminish, and the resource variables become cost-effective.

Clearly, the assumptions seeding the model are critical, and current research is not a source for exact answers. Nevertheless, the preponderance of evidence is in the direction of school effectiveness being a substantial determinant of achievement, and the model addresses this effectiveness by giving clues as to where to look. It must be stressed once again: This optimization model does not give a policy answer. In essence, it is a decision support system, or a calculation machine providing results based on the user-defined assumptions. While the optimization process will mathematically provide the best solution, the solutions may not be compatible with perceptions of the situation.

This being said, some broad principles do apply. Because the model is a simulation asking "what if" questions, the principles are in terms of "what if":

- What if the parameters in the illustration were valid?
  o The potential gain in achievement is substantial, most of which is associated with the effectiveness variables.
- What if the class size effect size is set to the value estimated from the STAR experiment (.1)?
  o There would be no change in the conclusion. The effectiveness variables are still more cost-effective. The effect size for the class size variable would still be smaller, and the incremental costs would be higher compared to the effectiveness variables.
- What if the actual achievement for the school were different?
  o Remember, the prediction formula must predict the actual achievement for the school in question. To achieve this equalization, an effectiveness factor (C) is introduced indicating how effective the school is. If the actual achievement is higher than predicted, then the school is more effective in implementation.[21]
- What if the target amount is changed?
  o As the target amount increases, so does the predicted achievement, but at a decelerating rate--the benefits gradually get smaller. Various predicted achievement levels for various funding targets: $50,000 = 20.78; $100,000 = 23.82; $150,000 = 24.75. As the school becomes more effective, the potential achievement gain diminished.

At first appearance, the model seems to treat each variable as being independent when in reality it is more likely that the variables work in combination. Achievement results are due to a combination of efforts, with resource and effectiveness policies working together. The staffing options can be effective only if clear directions regarding behaviors are provided. An obvious example is: If

the goal is to improve music achievement, hire a music teacher and provide a clear set of expectations. While the illustration emphasizes the policies at the school level, surely district wide policies are also influential. In that vein, it is possible and maybe wise to have a highly skilled staff member provide service to more than one school building.

There are an infinite number of possibilities, so only the major points will be reported here. First, the incremental cost parameters are reasonably accurate, and the incremental costs for the effectiveness variables are most likely less than those for the resource variables. Second, changes in effect size and initial conditions must be substantial before there will be a change in the optimization results. Third, the resource variables become cost-effective only when the effectiveness variables are near the maximum, and that happens only when the actual achievement is substantially higher than the predicted achievement.

These results have consequences for the research reviewed in the earlier article in that it changes the research question. No longer are the questions, does class size make a difference, or how much of a difference does it make? The new question is: Under what set of policy and behavioral conditions does achievement improve, and by how much?

The Policy Development Process

Most importantly, the optimization model is an iterative process. Once the result for one set of policy options is developed, it must be evaluated and refined. If a particular set of policy options is unworkable, setting a variable constraint to a different level modifies outcomes. As policy options are narrowed, so is the target cost, bringing the policy analysis to a desired funding level.

In reality, the results are only as good as the assumptions, so at every step of the process the assumptions must be evaluated. In other words, the model is a tester of assumptions, or a tester of the relationships among policies, behaviors, and achievement. As such, the best policy scenario is most likely natural rather than unnatural, with a sense of beauty or elegance rather than complexity.

While Solver refers to the various policy options as scenarios, these are really various theories of achievement production. In some cases, there is research defining the characteristics and estimating the effect size, but in many cases the relationship between the policy, behavior, and achievement outcomes is common sense. Here is an illustration of an actual linkage between policy, behavior, and achievement. In the early 1970s when our daughter attended the Shaker Heights, Ohio school system, the board of education adopted a reading and writing policy applicable to all students, teachers, and families. Starting in the fourth grade, every student was required to read a book of their choosing every week and prepare a written summary based on a prescribed outline. The student's family was required to enforce the policy at home, inspect the written summary, and attest to its authenticity. Finally, teachers were required to review the summary and judge whether it met the prescribed standard. If not, the report had to be redone. Reading and writing achievement improved. No research study was required.

This example emphasizes a theory of time-on-task; that is, the more time spent on an activity, the greater the performance. This is a possible scenario for inclusion in a policy analysis optimization by estimating the effect size and incremental cost. There are many other possibilities too numerous to fully discuss here, but the work of Levin, Walberg, and those mentioned in earlier articles in this

issue are starting points. Each school will have to critically evaluate their performance and decide what are the most pressing issues to address. Again, there is no single solution to all problems.

While many people think SES is the best predictor of student achievement, this is not the case. The best predictor of achievement is whether the student received instruction in the subject matter included in the achievement test. Students who have had a class in algebra consistently perform better on an algebra test than students who did not. Unfortunately, data for the effectiveness variables are limited, and the shortcoming must be compensated for by stringent analysis. Educators with expertise in several specialties—curriculum and instruction, administration, finance, social foundations—should bring their expertise to bear in analyzing each possible scenario. In this search, each school must do its own critique, answering the following questions:

- What are the appropriate outcome goals?
- What are the best educational practices?
- Where does the school stand in relationship to best practices?
- Are there model schools to emulate?
- What policies will most influence the desired behaviors of instructional staff, students, families, and, when possible, the community?
- What is the process to assign and monitor behavior with regard to training, written clarification, individual assistance, progress reports, evaluation, and rewards for success?
- What financial resources are required for additional staff, the purchase of additional time from existing staff, instructional materials, and specialized facilities?
- What is the estimated effect size to be accrued from the implementation of the policy?
- What is the feasibility of an effective implementation?

After the possible policy scenarios are developed, they can be entered into the optimization model where alternatives are evaluated by estimating the respective potential achievement gains. Instead of relying on opinion or on a review of the research literature, this policy development model requires a clear and comprehensive statement of the alternatives followed by a critical and comparative evaluation of the alternatives based on cost and potential benefits.

Other Considerations:
General Principles of the Optimization Model

There are other techniques to make the model more sophisticated:

- It is possible and even desirable to set boundaries for the policy variables. The boundaries consist of maximum and minimum levels, which the optimization process cannot exceed.
- Boundaries can be set so that one variable with a positive slope can be limited in order that another variable can be increased.
- It is possible to include policy variables with negative slopes, measuring the potential gain from reducing costs in these areas and applying the funds to another more productive area. These are called opportunity costs.
- It is possible to include non-achievement goals in the model as long as there is a measure of attainment, a

measure of the initial conditions, estimated costs, and estimated effect sizes.

Solver creates several reports to assist in the analysis of the scenario. The "Sensitivity Report" contains information demonstrating how sensitive a solution is to changes in the formulas used in the scenario. It measures the increase in the predicted achievement level for a unit change in each of the determinants and constraints. The "Answer Report" provides the predicted achievement level; the original and final values of the determinants; and information about the constraints. The "Limits Report" lists the achievement levels and the determinants with their values, and lower and upper limits.

Value of a Policy Analysis Simulation

Building a simulation model has several potential benefits:[22] The exercise of building a simulation model often reveals structures and relationships not previously apparent. As a result, there is a greater understanding of the complex process of achievement production. The modeling process can identify areas where additional research is needed. Having built a model, it is possible to analyze it mathematically to help suggest courses of action not otherwise apparent. Experimentation with many options is possible with a model whereas it is often not possible or desirable to experiment on the actual situation. Many policy options can be tested, first separating practical from impractical solutions. If a satisfactory policy option is identified during the simulation process, it gives clear directions as to how it could be implemented and tested in an actual situation. As more experience and knowledge is gained, the model is enhanced.

When decisions are made based on opinion, the underlying assumptions regarding policy actions, costs, and predicted benefits are mostly ambiguous; therefore, there is no method to test the likelihood of achieving the desired goals. While productivity research may give some helpful direction, research in and of itself does not provide sufficient information regarding particular situations (policy actions and costs) to accurately predict outcomes. Only through a comprehensive policy analysis model can the underlying assumptions be clearly stated, evaluated, and tested.

A Final Word

In the early 1900s, the notion of gravity took a major turn. Einstein developed his theories of general and special relativity based on the idea that space is actually curved—nonlinear. Years later, the theory was confirmed by experiment showing that light from distant stars indeed curves around the sun on the way to earth. Space travel is calculated by his equations. While not of the same magnitude, it is reasonable that the relationship between achievement and policy variables is better explained by a nonlinear function, and it is worthy to test by experiment. After all, there are no experiments demonstrating that the relation is linear!

Admittedly an exaggeration, here is a characterization between the effective and noneffective method of allocating of resources. This first is called the "Professor Henry Hill" method after the lead character in the Meredith Wilson musical, "Music Man." Hill, a traveling salesman, convinced the people of River City to purchase from him bright new uniforms with shiny buttons for the school band, and in return he could make beautiful music solving all the "troubles here in River City." Once he got the money, he employed the "think method" of instruction. If the students would "think" how nice it would be to march down the street in their magnificent

uniforms with their parents and community cheering them on, they would be able to skillfully play their instruments. Sure enough, it worked and everyone was treated to a magnificent parade with "Seventy-six Trombones."

The second example is called the "Carnegie Hall" method after a common musician's joke. While walking down the streets of New York City, a person asked a stranger, "How do you get to Carnegie Hall?" The stranger replied, "Practice, practice, practice." Imagine a situation where students are in an instrumental music class learning to play an instrument. They meet regularly, receive structured and competent instruction, take their instrument home, and the parents oversee thirty minutes of practice every day. At each step, there is a clear policy directing student behavior. It does not take a sophisticated research study to determine the difference of musical quality being produced by the two paradigms.

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Endnotes

[1] In particular, the previous article, "A Practical Method of Policy Analysis by Estimating Effect Size," led to a number of underlying assumptions that will guide the analysis here. See Appendix A for a list of these.

[2] The current achievement theories and models tend to follow the interpretation of the physical science laws: If one variable changes, the consequences are automatic. If students leave the classroom, does the knowledge of the remaining students increase automatically and at the speed of light? Do teachers and students, like "mother nature," automatically know what to do, or must another process take place?

[3] All subsequent references to Glass and Smith in this article refer to Gene V. Glass and Mary Lee Smith, *Meta-Analysis of Research on the Relationship of Class-Size and Achievement* (San Francisco, CA: Far West Laboratory for Educational Research and Development, 1978).

[4] All subsequent references to Hedges et al. in this article refer to Larry V. Hedges, Richard D. Laine, and Rob Greenwald, "Does Money Matter? A Meta-Analysis of Studies of the Effects of Differential School Inputs on Student Outcomes," *Educational Researcher* 23 (April 1994): 5-14.

[5] Correspondingly, a substantial number of research studies openly state a purpose of proving Eric Hanushek, a critic of these types of studies, wrong! The same was true in the 1970s when Coleman et al. (1966) issued the report, *Equality of Educational Opportunity*, with a conclusion showing the substantial relationship between achievement and socioeconomic status and a smaller relationship with resources.

[6] All subsequent references to Levin in this article refer to Henry M. Levin, "Raising School Productivity: An X-Efficiency Approach," *Economics of Education Review* 16 (June 1997): 303-311.

[7] All subsequent references to Walberg in this article refer to Herbert J. Walberg, "Improving the Productivity of America's Schools," *Educational Leadership* 41 (May 1984): 19-27.

[8] There is also an error distribution, or residual, not shown.

[9] Notice the similarity in shape between the integral of the normal curve and the "learning curve."

[10] "Scenario" is the description used in the Microsoft software, to be discussed later.

[11] Any policy variable can be included in a scenario if the effect sizes and incremental costs can be estimated.

[12] Available, but not included in the illustration because of small effect size estimates and limited space.

[13] Available, but not included in the illustration because of small effect size estimates and limited space.

[14] See Achilles et al. (1993).

[15] The details are provided in Phelps (2008).

[16] While necessary for this policy analysis by policymakers and practitioners, reporting the status and progress of schools to the public is valuable as well. A comprehensive review of these issues is available in Phelps (2009).

[17] The staffing qualifications and instructional materials categories are omitted from the illustration because of limited space and their small effect sizes, but they could be included as resource variables in a full simulation. The organizational effectiveness category is also omitted because there are no estimates of effect size.

[18] The Z-scores are converted into percentiles, and the predicted achievement equation is made to equal actual achievement by determining the value of C.

[19] See Appendix B.

[20] The value of C is derived via Microsoft Excel Solver. The Target Cell is set to 100 (the Actual Achievement level), By CHANGING CELLS is the value of C.

[21] Various actual achievement values were entered with the corresponding C values: 80 = -.60; 100 = .26; 120 = 1.5.

[22] Hilary P. Williams, *Model Building in Mathematical Programming*, 2nd ed. (New York: John Wiley, 1985), 3.

Appendix A
Underlying Assumptions for the Policy Analysis Model

- The teacher/pupil ratio is a more appropriate policy measure of teacher concentration than is class size (pupil/teacher ratio).
- Influence of SES is critical in measuring the effect size of the teacher/pupil ratio.
- The evidence from the previous articles in this issue discounts the Glass and Smith proposition of increased marginal gains for class sizes under 15, so their proposition will not be included.
- The R2, a nonlinear measure of effect size, has distinct advantages over the other options for developing a comprehensive policy strategy.
- There is substantial collinearity among most educational variables and the estimated effect sizes depend on the attribution of the common variance. The effect size estimate varies depending on how the common variance is attributed. Therefore, a maximum to minimum range is an appropriate estimate.
- Because of the substantial collinearity, it is best to combine the instructional variables into conceptual and statistical categories and estimate the effect size of the entire category.
- It is likely that the instructional and organizational variables work cooperatively with the resource variables.
- Some schools are more effective in implementing the policy options. If more attention is paid to the implementation, it is possible to achieve more than the predicted gain based on resource level alone.

Appendix B
Logistic Growth Curve and Calculation Formulaes

Logistic Growth Curve
Logistic growth: Rate of growth is proportional to the amount present and to the difference between the amount present and a fixed amount (Barnett and Ziegler 1984, 819).

$dy/dt = ky(M-y)$ with $k, t > 0$

where

k = rate

M = maximum

or

$y = M / 1 + ce{-Mt}$

Calculation Formulas
In Cartesian geometry, the origin of a graph is the intersection of the X- and Y-axes. This is the case with standard or Z-scores at point X = Zero and Y = Zero. The origin of the graph changes when Z-scores are transformed into percentiles. Because the mean (50th percentile) of the explanatory variable is equal to the mean of the achievement variable, the origin of the percentile graph is at the 50th percentile; and because the normal curve is symmetrical above and below this point, half of the distribution is above, and half is below. Finally, when the explanatory variable is a zero Z-score or the coefficient is zero, then the achievement variable is at the mean or 50th percentile.

Achievement is calculated from the percentile position of the school and the effect size, the R^2. The initial condition determines the percentile position for the actual achievement and the optimal condition for the predicted achievement.

- The contribution each variable makes to achievement is calculated from the percentile position and the R^2. The percentile position is calculated from the initial or optimal condition Z-score by the Excel function, NORMSDIST:

    Percentile = NORMSDIST (z)

- Because a policy variable at the mean predicts achievement at the mean, the calculations are the contributions to achievement above or below the 50th percentile.
- To calculate the contribution (the difference from the mean), .50 is subtracted from the percentile and multiplied by the R^2:

    $\Delta = (Ptile -.5) * R^2$

- The contributions made by the variables, the Δ's, are summed. Because these are measures above and below the mean, .50 must be added to the sum of the individual contributions to obtain the predicted achievement level:

    $PA = \sum \Delta + .50$