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The Information-Theoretic Approach to Model Selection: Description and Case Study

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ABSTRACT

In the wildlife literature there has been some recent criticism of statistical significance testing. In the past few years, both the Journal of Wildlife Management and the Wildlife Society Bulletin have published articles criticizing the overuse and misuse of hypothesis tests. One alternative to using hypothesis tests for model selection is the information-theoretic approach, proposed by Burnham and Anderson (1998). This technique uses values such as the Akaike Information Criterion and others to choose a set of plausible models from a set of *a priori* candidate models. Inferences are based on the set of plausible models, rather than on a single selected best model, and model-averaged point estimates of parameters may be used for prediction. The Burnham and Anderson method is gaining popularity in the wildlife science community, and statisticians who work with wildlife scientists should be aware of this analysis technique and how to use it properly. This paper will introduce statisticians to the information-theoretic approach to model selection and the statistical theory underlying it, as well as demonstrate the technique using data on bird species richness and abundance in riparian areas in southeastern Nebraska.

KEY WORDS: Akaike information criterion, Schwarz's Bayesian Information Criterion, Kullback-Liebler distance.

1 Introduction

Recently in the wildlife literature, several articles have appeared criticizing the use of statistical significance testing. These papers, appearing in the *Wildlife Society Bulletin* and *The Journal of Wildlife Management*, focus on the overuse and misuse of hypothesis tests. Cherry (1998) counted the number of p-values that appeared in the 1995 issue of *The Journal of Wildlife Management*, and found more than 2,400. He uses this large number as evidence of the overuse of tests, and notes that criticism of statistical testing has appeared in many disciplines, including statistics itself. Wildlife, it seems, is merely the latest discipline to note the widespread misuse of testing. Using quotes from Frank Yates, D. R. Cox and John Nelder, Cherry (1998) presents four major problem areas that he believes has led to this proliferation of inappropriate p-values. They include unnecessary testing, confusion over power analyses, misunderstanding the assumptions of hypothesis tests, and fixed-level tests.

These are the same problem areas discussed by Johnson (1999). Johnson blames the overuse and misuse of statistical tools in the wildlife field on the increased emphasis on hypothesis testing and power analyses. He gives various incorrect ways scientists interpret p-values, and contends that the correct interpretation of the p-value depends on assumptions that are usually questionable. In particular, he questions the calculation of the p-value given a true null hypothesis. He argues that point null hypotheses are “almost invariably known to be false before any data are collected” (Johnson 1999) and this leads to “gratuitous” significance testing. Johnson does note that while point null hypotheses are nearly always false in observational studies, they are often reasonable for designed experiments.

It is the distinction between observational studies and designed experiments that is the focus of other articles in this area. Anderson et al. (2000) contend that the paradigm of significance testing is not wrong, but simply that it is not informative. They also note that significance testing is not particularly useful in model or variable selection. The importance of model and variable selection is perhaps the most important distinction between the analyses of observation studies and designed experiments. Most wildlife studies are observational, and the analysis of such studies tends to focus on selecting the set of predictor variables which best explain the variation in the response variable. Cherry (1998) and Anderson et al. (2001) both point out that testing in regression settings tends to focus on testing

whether a relationship between the predictor and response variable exists. The question of interest, however, is usually not whether a relationship exists, but what kind of relationship exists. Cherry (1998), Thompson (1999), and Anderson et al. (2000) all contend that this question can best be answered by estimation, rather than testing. Anderson et al. (2000) propose an alternative to null hypothesis testing that incorporates both model selection and estimation. This method, termed the information-theoretic approach, bases model selection on information criteria and estimates effects using model averaging.

The information-theoretic approach, detailed in Burnham and Anderson (1992, 1998) and Anderson et al. (2000, 2001), has been gaining popularity in the wildlife literature as an analysis method. The purpose of this paper is to introduce consulting statisticians to the information-theoretic approach. This includes both the statistical theory underlying the approach, as well as how the method can be practically used for data analysis. The use of the information-theoretic method will be demonstrated using data on bird species richness and abundance in riparian areas in southeastern Nebraska. Finally, some criticisms and drawbacks of the information-theoretic approach will be presented.

2 The Information-Theoretic Approach: Theory

Burnham and Anderson (1998) give a detailed description of the statistical theory behind the information theoretic method. This section presents an overview of the arguments described in Burnham and Anderson (1998).

The goal of the information-theoretic approach to model selection is finding the best model to describe the relationship between a set of predictor variables and a response variable. It is based on the concept of minimizing the distance between two models. Burnham and Anderson (1998) describe how Kullback and Leibler derived a measure of the discrepancy between two models, f and g . This measurement, called the Kullback-Leibler (K-L) distance, measures the discrepancy between the “truth” (f) and an approximating model (g). The smaller the K-L distance, the closer the approximating model g comes to the truth (f). The approximating model g depends on parameters θ , and so is written $g(x|\theta)$. The “truth” depends only on the data, $f(x)$. The K-L distance between models f and g is defined

as

$$I(f, g) = \int f(x) \log \left(\frac{f(x)}{g(x|\theta)} \right) dx, \quad (1)$$

where f and g are continuous functions. Anderson et al. (2000) define $I(f, g)$ as the “information” lost when the truth f is approximated by g . The goal of model selection is then to minimize $I(f, g)$ over g . If $I(f, g)$ is to be minimized over g , then the K-L distance must be calculated for various approximating models g . But, f is “truth,” which is unknown. To see how the K-L distance may be used with an unknown f , Burnham and Anderson (1998) and Anderson et al. (2000) write $I(f, g)$ in a different way:

$$I(f, g) = \int f(x) \log \left(\frac{f(x)}{g(x|\theta)} \right) dx \quad (2)$$

$$= \int f(x) \log(f(x)) dx - \int f(x) \log(g(x|\theta)) dx \quad (3)$$

$$= E_f[\log(f(x))] - E_f[\log(g(x|\theta))] \quad (4)$$

$$= C - E_f[\log(g(x|\theta))] \quad (5)$$

$$I(f, g) - C = -E_f[\log(g(x|\theta))] \quad (6)$$

where C is a constant that is fixed across all approximating models g . The quantity $I(f, g) - C$ is the relative K-L distance and $E_f[\log(g(x|\theta))]$ is therefore the focus in model selection (Anderson et al. 2000). The basic principle of the information-theoretic approach is to postulate an *a priori* set of candidate models $g_i(x|\theta)$ and select the best of these as the model that minimizes $E_f[\log(g(x|\theta))]$.

This quantity, $E_f[\log(g(x|\theta))]$, cannot be calculated because the parameters θ are unknown. Instead, an estimate of the relative K-L distance between f and g_i would help select the best of the candidate models. This means that model selection could be based on minimizing the expected relative K-L distance (Anderson et al. 2000).

Burnham and Anderson (1998) describe how Akaike, in 1973, found an estimator of the expected relative K-L distance based on a bias corrected maximized log-likelihood value—the Akaike Information Criterion (AIC):

$$AIC = -2 \log(l(\hat{\theta})) + 2p \quad (7)$$

where p is the number of estimated parameters in the approximating model, g . The approximating model with the smallest AIC value is considered to be “closest” to the truth (Burnham and Anderson 1998, Anderson et al. 2000).

Burnham and Anderson (1998) and Anderson et al. (2000, 2001) emphasize that if there are too many parameters p with respect to the sample size n , then AIC will perform poorly. Instead, a modified criterion, AIC_c , should be used

$$AIC_c = -2\log(l(\hat{\theta})) + 2p \left(\frac{n}{n-p-1} \right) \quad (8)$$

$$= AIC + \frac{2p(p+1)}{n-p-1} \quad (9)$$

whenever $n/p < 40$. Anderson et al. (2001) believe that the use of AIC when AIC_c is more appropriate is a very common mistake when using the information-theoretic method.

Another modified criterion, $QAIC_c$, has been developed for over-dispersed count data.

$$QAIC_c = - \left[\frac{2\log(l(\hat{\theta}))}{\hat{c}} \right] + 2p + \frac{2p(p+1)}{n-p-1}, \quad (10)$$

where $\hat{c}$ is the variance inflation factor (Burnham and Anderson 1998; Anderson et al. 2000, 2001).

Whichever model selection criterion is chosen, the same criterion should be used for all candidate models in the set. The candidate model which minimizes the appropriate criterion is selected as the “best” model, and is used as the basis for data analysis and inference (Burnham and Anderson 1998).

One final note on information criteria. Burnham and Anderson (1998) also discuss information criteria that are “dimension consistent.” One such criterion is Schwarz’s Bayesian Information Criterion, BIC:

$$BIC = -2\log(l(\hat{\theta})) + p\log(n). \quad (11)$$

“Dimension consistent” criteria such as this one are based on the assumptions that an exactly “true” model exists, the “true” model is one of the candidate models, and the goal of model selection is to select the “true” model. Burnham and Anderson (1998) emphasize that these “dimension consistent” criteria are not estimates of the K-L distance, and are not recommended.

3 The Information-Theoretic Approach: Application

An important tenet of the information-theoretic approach is selection of a set of *a priori* models based on careful thought about the relationships between the independent and dependent variables. Common ways to select the candidate models are using published literature and subject-matter knowledge in evaluating and selecting the independent variables in relation to the dependent variable in question. Once an *a priori* set of candidate models has been selected, and the appropriate information criterion has been calculated for all candidate models, the analysis question becomes one of model selection and inference on the selected model should be accomplished. Because AIC (as well as AIC_c and $QAIC_c$) is a relative measure, Burnham and Anderson (1998) recommend calculating the AIC differences

$$\Delta_i = AIC_i - \min AIC \quad (12)$$

for each candidate model. This means that the “best” model will have $\Delta_i = 0$. Burnham and Anderson (1998) also recommend that these differences, not the AIC values, be displayed in publications. These differences allow for easy model comparisons and rankings. The larger the Δ_i , the less plausible the particular candidate model is “best” for data sets such as the one collected. Burnham and Anderson (1998) give some guidelines for interpretation of Δ_i values. If $\Delta_i \leq 2$, there is substantial support for the candidate model. If $4 < \Delta_i < 7$, there is considerably less support. If $\Delta_i > 10$, there is no support for the candidate model. Based on these Δ_i values, the researcher selects the best and other supporting models, and identifies any models that are clearly not supported. It is recommended that any model with $\Delta_i \leq 4$ be given consideration (Burnham and Anderson 1998).

Burnham and Anderson (1998) and Anderson et al. (2000) also recommend making some additional calculations to better quantify the probability of each model being the “best.” They show that a transformation of the Δ_i gives the likelihood of the model, given the data. This tranformation, recommended by Akaike as $\exp(-\frac{1}{2}\Delta_i)$, can be normalized so that the sum of the transformed Δ_i is 1. These normalized values, called the Akaike weights, are calculated as

$$w_i = \frac{\exp(-\frac{1}{2}\Delta_i)}{\sum_{r=1}^R \exp(-\frac{1}{2}\Delta_r)} \quad (13)$$

where $r = 1, 2, \dots, R$ and R identifies the number of models in the candidate model set. The weight w_i can be interpreted as the approximate probability that model i is the “best” in the set of candidate models. Also, the relative likelihood of model i versus model j can be calculated as w_i/w_j (Burnham and Anderson 1998; Anderson et al. 2000).

Once the selected model is chosen, inference may be undertaken. The analyst must remember that inference is conditional not only on the data set as usual, but also on the set of candidate models (Anderson et al. 2000). As a result of this conditioning on the set of the candidate models, the usual estimate of sampling variance must be adjusted by including a variance component to account for uncertainty in model selection. This adjustment ensures that estimates of precision are unconditional on the selected model, but are conditional on the candidate models in the set. Anderson et al. (2000) give an estimator of the unconditional variance for the parameter θ from the chosen model:

$$\widehat{\text{var}}(\widehat{\theta}) = \left[\sum_{i=1}^R w_i \sqrt{\widehat{\text{var}}(\widehat{\theta}_i | g_i) + (\widehat{\theta}_i - \widehat{\theta})^2} \right]^2 \quad (14)$$

where

$$\widehat{\theta} = \sum_{i=1}^R w_i \widehat{\theta}_i. \quad (15)$$

This variance estimator, attributed to Buckland et al. (1997), includes the conditional sampling variance given model g_i and a variance component for model uncertainty (Anderson et al. 2000). This unconditional variance can be used to construct the usual 95% confidence intervals for a parameter.

In some situations there will be no one clearly “best” model. Thus, instead of basing inference in a single “best” model, inference can be based on an entire set of models by using a model-averaged estimator for a parameter θ . The model-averaged estimator for θ is the weighted average previously shown:

$$\widehat{\theta} = \sum_{i=1}^R w_i \widehat{\theta}_i. \quad (16)$$

A model-averaged estimator often has the advantages of better precision and reduced bias compared to the estimate of the parameter from a single “best” model (Anderson et al. 2000). Burnham and Anderson (1998) point out that when model-averaging, or simply calculating

the estimated unconditional sampling variance for a parameter, the weights w_i must be re-normalized so that they sum to 1 for the subset of models that contain the parameters of interest.

4 Case Study

The information-theoretic approach to model selection will be illustrated using data from a study to determine the effects of landscape on bird species richness and abundance in riparian areas in southeastern Nebraska. This study was carried out by co-author Micah Perkins, a former graduate student in the School of Natural Resource Sciences at the University of Nebraska. The study was carried out in the summers of 1999 and 2000. In 1999, fifteen study sites in southeastern Nebraska were used and three additional sites were added in 2000 (Perkins 2001).

Study sites were linear strips of wooded riparian corridors 500 m in length and approximately 50 m wide. Each site was visited four times during each year, and bird species richness and abundance data were collected on each visit. Richness and abundance data were averaged over visit in each year. Local-scale measurements were made at each study site in each year. These included canopy cover and height, understory foliage volume, ground cover, stream measurements, and others. Landscape variables such as land cover, ratio of cropland to grassland, percentage of woody cover, and others were also measured on each site in each year. Details about data collection and a full list of measured variables may be found in Perkins (2001).

Dependent variables were woodland bird species abundance and richness, edge species abundance and richness, and abundance and presence of individual species. Independent variables were the local-scale and landscape measurements taken on each study site. Following a major tenet of the information-theoretic method, a set of *a priori* models was selected. These candidate models contained predictor variables thought to affect species group and individual species response variables. Thirty models were considered for each of the species group response variables. Anderson et al. (2001) recommend the number of candidate models (R) be relatively small and suggest that situations with more models than observations ($R > n$) be avoided. Because the Perkins (2001) data sets have $n = 33$ observations, $R = 30$

was the maximum number of models considered.

For the purpose of illustration of the information-theoretic approach, only the analyses for the response variable woodland species richness are presented. Analyses of the other response variables and discussion of the results are in Perkins (2001). For woodland species richness, 30 candidate models were considered. Each of these models has the form

$$y_{ij} = \text{year}_i + \beta_0 + \beta_1 \text{predvar}_{1ij} + \beta_2 \text{predvar}_{2ij} + \dots + \beta_k \text{predvar}_{kij} + \epsilon_{ij}, \quad (17)$$

where $i = 1, 2$, $j = 1, \dots, n_i$ ($n_1 = 15, n_2 = 18$) and k is the total number of predictor variables for the model. The year effect was assumed to be random. This means that for each candidate model, there are k predictor variable parameters, one intercept, and two variance components estimated. This gives a total of $p = k + 3$ for the penalty term in the appropriate information criterion. Because of the small sample size, AIC_c was used. The independent variables (Table 1) were used in 30 models, shown with their AIC_c , Δ_i values, and Akaike weights w_i (Table 2). The models are shown in increasing order of AIC_c values; however, it doesn't matter in which order the models are fit, nor does the order of variables matter.

The first five models in Table 2 have $\Delta_i < 4$, and are given consideration. Recall that w_i is interpreted as the approximate probability that model i is the K-L "best." The w_i for models in which a particular variable appears may be summed to provide an indication of the importance of that variable (Anderson et al. 2000). The variable p500 (percent woody cover out to 500 m) appears in 14 of the 30 models and has a sum of $w_i = 0.77803$. This variable also appears in four out of the five top models. The next most important variable is GCSoil (ground cover percentage in soil), with $\sum w_i = 0.46277$.

Anderson et al. (2000) also point out that w_i/w_j is the relative likelihood of model i versus model j . Comparing the first and second models gives $w_1/w_2 = 0.37123/0.21418 = 1.733$. This implies that model 1 is not clearly superior to model 2, and rather than ignore model selection uncertainty, the model-averaged estimates of the effects and their unconditional standard errors were calculated. The 14 models containing the p500 variable, the individual model estimates of the p500 regression coefficient ($\hat{\beta}_{\text{p500}}$), conditional standard errors ($\text{se}(\hat{\beta}_{\text{p500}}|g_i)$), and re-scaled w_i , are given in Table 3. The first step is to find the model-averaged estimate of the effect of p500. This is simply the weighted average of the 14

individual estimates of the p500 effect.

$$\widehat{\beta}_{p500} = \sum_{i=1}^{14} w_i \widehat{\beta}_{p500i} = (0.47713)(0.04350) + \dots + (0.00000)(0.07609) = 0.05773 \quad (18)$$

The next step is to calculate the unconditional standard error for the p500 effect. The unconditional variance for the p500 effect is

$$\text{var}(\widehat{\beta}_{p500}) = \left[\sum_{i=1}^{14} w_i \sqrt{\text{var}(\widehat{\beta}_{p500i} | g_i) + (\widehat{\beta}_{p500i} - \widehat{\beta}_{p500})^2} \right]^2 \quad (19)$$

$$= \left[(0.47713) \sqrt{(0.01400)^2 + (0.04350 - 0.05773)^2} \right. \quad (20)$$

$$\left. + (0.18453) \sqrt{(0.01578)^2 + (0.07859 - 0.05773)^2} + \dots \right. \quad (21)$$

$$\left. + (0.00000) \sqrt{(0.01635)^2 + (0.06509 - 0.05773)^2} \right]^2 \quad (22)$$

$$= 0.00045484. \quad (23)$$

The unconditional standard error for the p500 effect is therefore $\sqrt{0.00045484} = 0.021327$. An approximate 95% confidence interval for the effect of p500 on woodland species richness would then be $\widehat{\beta}_{p500} \pm 2\text{se}(\widehat{\beta}_{p500})$, or 0.05773 ± 0.04265 .

Model-averaged estimates and unconditional standard errors could be found similarly for all of the predictor variables in the best models.

To compare the information-theoretic approach to the standard approach, the stepwise regression method was also used to determine which subset of the variables in Table 1 best explain the variation in woodland species richness. However, due to the limitations of PROC REG, the random year effect included in the 30 candidate models was not included in the stepwise model. The first three variables selected by the stepwise procedure were p500, GCSoil and GCLitter, the same variables in the minimum AIC_c model. The stepwise procedure went on to select a1000, C3, ufv12, and a500 as well. During the course of the procedure, p500 and GCSoil were removed from the model. The information-theoretic approach selected these two variables as the most important, with $\sum w_i = 0.77803$ and 0.46277 , respectively. So, while the initial variables chosen by the stepwise procedure are the same as those selected by the information-theoretic approach, the results from the two approaches are quite different.

5 Conclusion

The information-theoretic approach to model selection is becoming more widely used in the wildlife science literature. The advantages of the approach include its ability to compare non-nested models and its emphasis on *a priori* thought rather than data dredging. There have been some criticisms of the method, however. Guthery et al. (2001) point out that likelihood based methods are by definition parametric. If an incorrect probability distribution is assumed for the data, the inference may not be valid. Guthery et al. (2001) also worry that the current rote approach to significance testing will simply be replaced by a rote calculation of information criteria.

There are many situations in which the information-theoretic approach to model selection is inappropriate. It is not meant to replace hypothesis testing in designed experiments. In formal experimentation, specific predictor variables are chosen and their effects can be validly assessed by testing specific *a priori* null and alternative hypotheses. In observational studies, however, analyses tend to focus on model selection (i.e., which of the myriad variables observed are useful in explaining the response). In this situation, the information-theoretic method assists the scientist in model selection while avoiding significance tests for clearly false null hypotheses. Moreover, the method is not difficult to apply and can be approached using standard statistical software.

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Table 1: Woodland Species Richness: Abbreviations for Independent Variables

Abbreviation	Description
p500	percent woody cover out to 500 m
p1000	percent woody cover out to 1000 m
p2000	percent woody cover out to 2000 m
a500	area weighted shape index out to 500 m
a1000	area weighted shape index out to 1000 m
a2000	area weighted shape index out to 2000 m
areaha	area of study site
GCSoil	ground cover percentage in soil
GCLitter	ground cover percentage in litter
GCGrass	ground cover percentage in grass
GCWoody	ground cover percentage in woody growth
GCForb	ground cover percentage in forbs
ufv03	understory foliage volume 0 - 0.3 m
ufv031	understory foliage volume 0.3 - 1 m
ufv12	understory foliage volume 1 - 2 m
ufv23	understory foliage volume 2 - 3 m
dsd	deciduous shrub density
ch	canopy height
snag	number of standing dead trees
r500	ratio of crop land to grass land out to 500 m
C1	trees 3 - 8 cm dbh
C2	trees 8 - 15 cm dbh
C3	trees 15 - 23 cm dbh
C4	trees 23 - 38 cm dbh
C5	trees 38 - 53 cm dbh
C6	trees > 53 cm dbh

Table 2: Woodland Species Richness: Candidate Models, AIC_c , Δ_i , and w_i

Independent Variables	AIC_c	Δ_i	w_i
p500, GCSoil, GCLitter	80.9	0	0.37123
ufv03, ufv031, ufv12, ufv23	82.0	1.1	0.21418
p500, GCGrass, GCWoody, ufv03	82.8	1.9	0.14357
p500, GCGrass, GCWoody	83.2	2.3	0.11754
p500, GCSoil	83.7	2.8	0.09154
p500, GCLitter	87.3	6.4	0.01513
p500, GCWoody	87.3	6.4	0.01513
p500, ufv03, ufv031	87.7	6.8	0.01239
p500, GCGrass	88.2	7.3	0.00965
ufv03, ufv031	88.8	7.9	0.00715
p500, ufv03	92.5	11.6	0.00112
ufv03, GCGrass	93.8	12.9	0.00059
p500, GCForb	94.2	13.3	0.00048
p500, areaha	96.2	15.3	0.00018
p500, a500	98.5	17.6	0.00006
p1000, areaha	99.6	18.7	0.00003
p1000, a1000	101.2	20.3	0.00001
p500, a500, areaha	101.2	20.3	0.00001
p1000, a1000, areaha	104.1	23.2	0.00000
p2000, areaha	106.0	25.1	0.00000
p500, r500	107.0	26.1	0.00000
p2000, a2000	107.7	26.8	0.00000
ch, snag	108.0	27.1	0.00000
ufv03, ufv031, ufv12, ufv23, dsd, ch	108.7	27.8	0.00000
dsd, ufv03, ufv031	108.7	27.8	0.00000
p2000, a2000, areaha	109.3	28.4	0.00000
ch, snag, dsd, ufv03, ufv031	115.4	34.5	0.00000
C4, C5, C6	119.5	38.6	0.00000
C4, C5, C6, g500	122.5	41.6	0.00000
C1, C2, C3, C4, C5, C6	141.2	60.3	0.00000

Table 3: Woodland Species Richness: 14 Models containing p500, estimates of $\widehat{\beta}_{p500}$, $se(\widehat{\beta}_{p500}|g_i)$ and re-scaled w_i

Model	$\widehat{\beta}_{p500}$	$se(\widehat{\beta}_{p500} g_i)$	w_i
p500, GCSoil, GCLitter	0.04350	0.01400	0.47713
p500, GCGrass, GCWoody, ufv03	0.07859	0.01578	0.18453
p500, GCGrass, GCWoody	0.07807	0.01543	0.15108
p500, GCSoil	0.05356	0.01328	0.11766
p500, GCLitter	0.04671	0.01511	0.01945
p500, GCWoody	0.08711	0.01658	0.01945
p500, ufv03, ufv031	0.05375	0.01523	0.01592
p500, GCGrass	0.06108	0.01311	0.01240
p500, ufv03	0.06845	0.01529	0.00144
p500, GCForb	0.06678	0.01456	0.00062
p500, areaha	0.05933	0.01577	0.00023
p500, a500	0.06378	0.01526	0.00007
p500, a500, areaha	0.05900	0.01611	0.00002
p500, r500	0.06509	0.01635	0.00000