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ON MULTIVARIATE ANALYSES OF CROSSOVER DESIGNS
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In crossover experiments, treatments are assigned to experimental units in successive periods. Traditional analyses of crossover designs with three or more periods assume that the observations in successive periods satisfy conditions similar to those utilized in the analysis of many repeated measures experiments. The successive measurements are assumed to satisfy conditions known as the Huynh-Feldt conditions. This paper gives a test for the Huynh-Feldt conditions and discusses possible analyses of crossover experiments, including tests for carryover, when the Huynh-Feldt conditions are not satisfied.

1. Introduction

Crossover experiments are special types of repeated measures experiments where the treatments being given to an experimental unit change over time. This paper has nothing to add to traditional analysis methods for two period crossover designs, and considers only those crossover designs which involve three or more periods.

Huynh and Feldt (1970) gave conditions under which repeated measures experiments can be analyzed in the same way that split plot experiments are analyzed. These conditions have since been called the Huynh-Feldt (H-F) conditions. If one has p repeated measures and if one lets Σ represent the variance-covariance matrix of the repeated measures on a randomly selected experimental unit, the H-F conditions are said to be satisfied if there exists a constant η and a p x 1 vector γ such that Σ = ηI_p + γj_p' + j_pγ'.

A test for the H-F conditions is obtained by testing $PΣP'$ = ηI_p-1 for some η where P is any p-1 x p matrix whose rows consist of orthogonal normalized contrasts. A test of whether a covariance matrix is a multiple of an identity matrix is usually called a test of sphericity in multivariate literature, and such a test is discussed in most multivariate methods books. The test is also described in the next section.

When the H-F conditions are not satisfied, there have been several alternative suggestions for analyzing repeated measures designs. Some suggestions involve making adjustments to the degrees of freedom associated with test statistics involving the repeated measures. There are two common adjustments, one given by Huynh and Feldt (1970) and one given by Greenhouse and Geisser (1959). Both of these adjustments reduce the degrees of freedom of ANOVA test statistics by multiplying their numerator and denominator degrees of freedom by an adjustment or correction factor.
A third method for analyzing repeated measures experiments, which is likely the most general approach, is to treat the vector of repeated measures as a multivariate response vector and apply multivariate analysis of variance methods to test the relevant hypotheses.

The above approaches have rarely, if ever, been applied to crossover designs in the published literature. In the next section, a test for the H-F conditions in a crossover design is given, and in the following sections, methods for analyzing crossover designs when the H-F conditions are not satisfied are proposed.

2. Testing for the H-F Conditions

Suppose a researcher has a crossover design with treatments given to experimental units in s different sequences where each sequence involves p periods. A traditional model (without carryover) for this setup is

\[ y_{ij} = \mu + S_i + \delta_{it} + T_j + P_k + \epsilon_{ijkl} \]

for \( i = 1, 2, \ldots, s; \ j = 1, 2, \ldots, p; \ k = 1, 2, \ldots, n_i \) where \( \mu \) represents an overall mean, \( S_i \) represents an effect due to the \( i \)th sequence, \( \delta_{it} \) represents an error which is associated with the \( \ell \)th subject in the \( i \)th sequence, \( T_j \) represents the effect of the \( j \)th treatment, \( P_k \) represents the effect of the \( k \)th period, and \( \epsilon_{ijkl} \) represents residual variation within the \( \ell \)th subject who received the \( j \)th treatment in the \( k \)th period of the \( i \)th sequence.

Let \( y_{il} \) be the \( p \times 1 \) vector of responses for the \( \ell \)th subject in the \( i \)th sequence and let \( \epsilon_{il} \) be the corresponding vector of errors. Let \( \Sigma = \text{Cov}[\epsilon_{il}] \), and assume the \( \epsilon_{il} \)'s are distributed independently and identically multivariate normal for \( i = 1, \ldots, s \) and \( \ell = 1, \ldots, n_i \). For convenience, let \( \mu_i = \text{E}[y_{il}] \) and note that the elements in the \( \mu_i \)'s are functions of the \( S_i \)'s, the \( T_j \)'s, and the \( P_k \)'s in model (1).

Let \( N \) be the total sample size, i.e., \( N = \sum_{i=1}^{s} n_i \). It is straightforward to show that

\[ \bar{\mu}_{i} = \frac{1}{n_i} \sum_{i=1}^{n_i} y_{il} \quad \text{and} \quad \Sigma = \frac{1}{N-s} \sum_{i=1}^{s} \sum_{l=1}^{n_i} (y_{il} - \mu_i)(y_{il} - \mu_i)' \]

are sufficient statistics for this problem.
It can be shown that \((N-s)\Sigma\) is distributed as a central Wishart distribution with \(N-s\) degrees and variance-covariance matrix \(\Sigma\) and that the \(\mu_i's\) are distributed independent \(N(\mu_i,(1/n_i)\Sigma), i=1,2,\ldots,s,\) Also the \(\mu_i's\) are independent of \(\Sigma\).

Let \(P\) be any \(p-1\times p\) matrix whose rows are orthogonal normalized contrasts and \(W=(N-s)P\Sigma P'\) then a test of the H-F conditions is based on \(\Delta=\frac{|W|}{p-1^{tr(W)^{p-1}}}\). A formula for approximating a p-value for this test statistic is given by Srivastava and Carter (1979, p. 327).

A SAS Analysis

A test for the H-F conditions in crossover designs can be easily obtained in SAS-GLM by using the REPEATED option with the following SAS commands where \(p\) represents the number of periods in each sequence and \(Y_1, Y_2, \ldots, Y_p\) represent the measurements taken in successive periods. In the SAS output, the test labeled as a test for sphericity is the test of the H-F conditions.

```
PROC GLM;
  CLASSES SEQUENCE;
  MODEL Y1--Yp = SEQUENCE;
  REPEATED PERIOD p POLYNOMIAL / PRINTE;
```

3. Alternative Analyses of Crossover Designs

In this section some different possible analyses of crossover designs are suggested for those situations where the H-F conditions are not satisfied.

3.1 Adjustments to the Degrees of Freedom

Greenhouse and Geisser (1959) and Huynh and Feldt (1970) suggested reductions in the degrees of freedom of the numerator and denominator mean squares of F ratios which involve time in
repeated measures experiments. These same kinds of adjustments can be made in crossover experiments.

Again let \( P \) be any \( p-1 \times p \) matrix whose rows are orthogonal normalized contrasts and let \( W = (N-s)P \Sigma P' \). Greenhouse and Geisser adjust the numerator and denominator degrees of freedom by

\[
\xi_1 = \frac{\left( \sum_{i=1}^{p-1} \sum_{j=1}^{p-1} w_{ij} \right)^2}{(p-1) \left( \sum_{i=1}^{p-1} \sum_{j=1}^{p-1} w_{ij} \right)}
\]

and Huynh and Feldt adjust the numerator and denominator degrees of freedom by

\[
\xi_2 = \frac{N(p-1) \xi_1 - 2}{(p-1)(N-s-(p-1) \xi_1)}
\]

If \( \xi_1 \) or \( \xi_2 \) should happen to be greater than 1, then they are replaced by 1. That is, the degrees of freedom associated with \( F \)-ratios are never increased.

### 3.2 A Multivariate Approach

Unfortunately, a multivariate analysis of crossover designs is not a straightforward generalization of a multivariate analysis of a repeated measures experiment. This is because the treatments are changing with respect to time in crossover experiments. That is, both time and treatments are changing in crossover experiments while only time is changing in repeated measures experiments.

To consider a multivariate approach to analyzing crossover experiments, once again let \( \mu_i \), \( \hat{\mu}_i \), and \( \Sigma \) be defined as they were in Section 2, and for illustration purposes consider model (1). For model (1)

\[
\mu_i = \begin{bmatrix} \mu_{i1} \\ \mu_{i2} \\ \vdots \\ \mu_{ip} \end{bmatrix} = \begin{bmatrix} \mu + S_i + T_{i1} + P_1 \\ \mu + S_i + T_{i2} + P_2 \\ \vdots \\ \mu + S_i + T_{ip} + P_p \end{bmatrix}, \ i=1,2,\ldots,s
\]

where \( T_{ki} \) represents the treatment assigned to an experimental unit in the kth period of the ith sequence, \( i=1,2,\ldots,s \), \( k=1,2,\ldots,p \).
Let $\beta' = [\mu S_1 S_2 \cdots S_s T_1 T_2 \cdots T_t P_1 P_2 \cdots P_p]$ be the vector of the parameters in model (1). Let $\mu' = [\mu_1' \mu_2' \cdots \mu_s']$. Thus $\mu$ is a $ps \times 1$ vector. Note that the elements of $\mu$ are all estimable functions and span the space of all estimable functions of $\beta$. Thus there exists a matrix $H$ such that $\mu = H\beta$.

Suppose $a'\beta$ is an estimable function of the parameters in $\beta$. Let $H^*$ be the Moore-Penrose generalized inverse of $H$. It can be shown that $a'\beta = a'H\mu = b'\mu$ where $b = H^*a$.

One unbiased estimator of $a'\beta$ which is based on the sufficient statistics is $b'\hat{\mu}$, and

$$b'\hat{\mu} \sim N(a'\beta, b'\Sigma b')$$

where

$$\Sigma' = \text{COV}(\hat{\mu}) = \begin{bmatrix} \frac{1}{n_1} \Sigma & 0 & \cdots & 0 \\ 0 & \frac{1}{n_2} \Sigma & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{n_s} \Sigma \end{bmatrix} = \Sigma \otimes \text{DIAG}(\frac{1}{n_1}, \frac{1}{n_2}, \ldots, \frac{1}{n_s}) .$$

Let $b' = [b_1' \ b_2' \ \cdots \ b_s']$ where each $b_i$ is a $p \times 1$ vector, then

$$b'\hat{\mu} = \sum_{i=1}^{s} b_i'\hat{\mu}_i \quad \text{and} \quad \text{VAR} (b'\hat{\mu}) = \sum_{i=1}^{s} \frac{1}{n_i} b_i'\Sigma b_i .$$

It must be noted that there may be other unbiased estimators of $a'\beta$ which depend on the sufficient statistics, so it cannot be guaranteed that the one given by $b'\hat{\mu}$ is the best. It should, however, be a PDG (pretty darn good) estimator of $a'\beta$.

Now let $V = \frac{b'\hat{\mu} - a'\beta}{\sqrt{\sum_{i=1}^{s} \frac{1}{n_i} b_i'\Sigma b_i}}$ . (2)

The result in (2) can be used to make inferences about $a'\beta$ if each of the samples sizes corresponding to each possible sequence of treatments is sufficiently large. In this case, one can assume the distribution of $V$ is approximately $N(0,1)$. But what can be done for small sample sizes?

For small sample sizes, one might try to approximate the
distribution of \( V \) with a \( t \) - distribution. Since the numerator and denominator of \( V \) are stochastically independent, one might try to use a Satterthwaite approximation to the degrees of freedom of \( V \).

By Satterthwaite's method, one would try to find \( u \) so that

\[
U = \frac{\sum_{i=1}^{g} \frac{1}{n_i} b_i' \Sigma b_i}{\sum_{i=1}^{g} \frac{1}{n_i}}
\]

is approximately distributed \( \chi^2(u) \) by equating the variance of \( U \) to \( 2u \), the variance of the chi-square distribution with \( u \) degrees of freedom, and solving for \( u \). However, at this point in time, the variance of \( U \) has not been obtained.

Since \( (N-s) \Sigma \) has a Wishart distribution with \( N-s \) degrees of freedom, one might conjecture that the degrees of freedom of \( V \) will be approximately equal to \( N-s \). We believe this to be a reasonable conjecture and, in fact, there appears to be some reason to believe that this is what Satterthwaite's method will eventually give. The evaluation of this conjecture has not yet been done.

### 3.3 A Mixed Models Approach

Consider a model for a crossover experiment which is based on the sufficient statistics. This model can be written as

\[
\hat{\mu} = H\beta + \varepsilon^*
\]

where \( \varepsilon^* \sim N(\mu, \Sigma^*) \). If \( \Sigma^* \) were known, the uniformly minimum variance unbiased estimator of an estimable function \( a'\beta \) is

\[
a'\beta_g \quad \text{where} \quad \beta_g = (H'\Sigma'^{-1}H)^{-1}H'\Sigma'^{-1}\hat{\mu}.
\]

In addition,

\[
a'\beta_g \sim N(a'\beta, a'(H'\Sigma'^{-1}H)^{-1}a).
\]

Unfortunately, \( \Sigma^* \) is unknown, but it can be estimated by

\[
\Sigma \otimes \text{diag} \left( \frac{1}{n_1}, \frac{1}{n_2}, \ldots, \frac{1}{n_s} \right).
\]

Then the estimated mixed model estimator of \( a'\beta \) is

\[
a'\beta_{eg} \quad \text{where} \quad \beta_{eg} = (H'\Sigma^{-1}H)^{-1}H'\Sigma^{-1}\hat{\mu}.
\]
Inferences about $a'b$ based on $a'b_{00}$ can be made using critical points from the standard normal distribution in those cases when all of the $n_i$'s are large and perhaps by using critical points from the $t$ - distribution with $N-s$ degrees of freedom when the sample sizes are small. The suitability of these approximations are in the process of being examined.

At this point in time, there exists no statistical software to carry out a multivariate analysis of crossover experiments (except to test for the H-F conditions). In the next section an example is given. SAS-IML has been used to carry out the multivariate analyses.

4. An Example

To illustrate the techniques discussed in the previous sections, consider the three period - three treatment crossover experiment discussed in Milliken and Johnson (1984). The design used in this experiment considered all possible sequences of the three treatments. This design produces an experimental design which is balanced for carry-over effects.

Table 1 shows the six sequences in the this data set.

<table>
<thead>
<tr>
<th>SEQUENCE</th>
<th>PERIOD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
</tr>
<tr>
<td>5</td>
<td>C</td>
</tr>
<tr>
<td>6</td>
<td>C</td>
</tr>
</tbody>
</table>

Let $\mu_{ij}$ represent the expected response for PERIOD $j$ in SEQUENCE $i$. The usual effects model parameters without carryover for the crossover design in Table 1 is:

\[
\begin{align*}
\mu_{11} &= \mu + p_1 + s_1 + \tau_1 \\
\mu_{21} &= \mu + p_1 + s_2 + \tau_1 \\
\mu_{31} &= \mu + p_1 + s_3 + \tau_2 \\
\mu_{41} &= \mu + p_1 + s_4 + \tau_2 \\
\mu_{51} &= \mu + p_1 + s_5 + \tau_3 \\
\mu_{61} &= \mu + p_1 + s_6 + \tau_3 \\
\mu_{12} &= \mu + p_2 + s_1 + \tau_1 \\
\mu_{22} &= \mu + p_2 + s_2 + \tau_1 \\
\mu_{32} &= \mu + p_2 + s_3 + \tau_2 \\
\mu_{42} &= \mu + p_2 + s_4 + \tau_2 \\
\mu_{52} &= \mu + p_2 + s_5 + \tau_3 \\
\mu_{62} &= \mu + p_2 + s_6 + \tau_3 \\
\mu_{13} &= \mu + p_3 + s_1 + \tau_1 \\
\mu_{23} &= \mu + p_3 + s_2 + \tau_1 \\
\mu_{33} &= \mu + p_3 + s_3 + \tau_2 \\
\mu_{43} &= \mu + p_3 + s_4 + \tau_2 \\
\mu_{53} &= \mu + p_3 + s_5 + \tau_3 \\
\mu_{63} &= \mu + p_3 + s_6 + \tau_3 \\
\end{align*}
\]

where $s_i$ represents the effect of the $i$th sequence, $i=1,2,3,4,5,6$, $T_j$ represents the effect of the $j$th treatment,
j=1,2,3, and $P_k$ represents the effect of the $k$th period, $k=1,2,3$.

Let $\beta' = [\mu_1 S_1 S_2 S_3 S_4 S_5 T_1 T_2 T_3 P_1 P_2 P_3]$, and
$\mu' = [\mu_1', \mu_2', \mu_3', \mu_4', \mu_5', \mu_6']$ where $\mu_{i'}=[\mu_{i1}, \mu_{i2}, \mu_{i3}]$ for $i=1,2,\ldots,6$.

For this example, the matrix $H$ which makes $\mu=H\beta$ is

$$
H = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
\end{bmatrix}.
$$

The SAS statements used to analyze the data in Milliken and Johnson and some of the results of the SAS analyses are shown in Appendix 1. The first set of analyses assume there are no unequal carryover effects; the second set of analyses test for unequal carryover effects and give comparisons between treatments in the presence of unequal carryover effects.

First a test for the H-F conditions is produced. This test is the same regardless of whether there is carryover or not. The results of this test are shown near the middle of page 4 of the SAS output. The test is labeled as a "Test for Sphericity." The test resulted in a $p$-value of 0.8775, and hence, the H-F conditions can not be rejected for this data. This is not surprising if one examines the correlation matrix shown on page 3 of the SAS output. The pairwise correlations between $Y_1$ and $Y_2$, $Y_1$ and $Y_3$, and $Y_2$ and $Y_3$ are 0.78, 0.80, and 0.74, respectively. These are nearly equal to one another, and the repeated measures seem to not only satisfy the H-F conditions, they also seem to possess compound symmetry.

The adjustment factors for the H-F and G-G adjustments to degrees of freedom are shown on page 6 of the SAS output. The
value of $\xi_1$ is 1.2378 and the value of $\xi_2$ is 0.9911. Since
the H-F factor is greater than 1, 1 would be used when making
adjustments to degrees of freedom using a H-F adjustment.
Nothing else on SAS output pages 1-6 are useful for our
purposes.

Since the H-F conditions are satisfied, one can make
inferences about the treatment effects from the analyses shown
on page 10. At the bottom of page 10, one finds estimates of
the treatment means as well as estimates, standard errors and
test statistics for making pairwise comparisons amongst the
treatments. Some of the output on page 10 has lines drawn
through it. We crossed these things out because SAS has not
computed these statistics correctly. Using methods discussed in
Chapter 28 of Milliken and Johnson (1984) one can compute
corrected standard errors for the treatment means. First one
must estimate the two variance components by solving
$$\theta^2_1 + 3\theta^2_2 = 10.2593 \text{ and } \theta^2_2 = 1.03876 \text{ for } \theta^2_1 \text{ and } \theta^2_2.$$ 
and $\theta^2_2 = 3.0735$. Then the standard error of each of the
treatment means is $\sqrt{(\theta^2_1 + \theta^2_2)/36} = .3380$. To construct a
confidence interval for the true treatment means one must use a
Satterthwaite approximation to compute an approximate degrees of
freedom for a t critical point.

If it were the case that the H-F conditions were not
satisfied, F-type ratios can be computed by squaring the t-
ratios, then the degrees of freedom corresponding to the
numerators and denominators of the F-ratios could be multiplied
by $\xi_1$ or $\xi_2$, and then finally, p-values could be recomputed.

Since the H-F conditions are satisfied for this data, and
the other analyses presented in this paper would not be
necessary. However, for illustration purposes, the other two
analyses are obtained by using the remaining SAS statements. On
page 12 of the SAS output, one finds the estimate of $\Sigma$. On page
13, estimates of the treatment means and their standard errors
are computed using the multivariate approach described in
Section 3.2. Confidence intervals for the true treatment means
are also given. Estimates of pairwise differences in the
treatments are shown along with their standard errors, t-tests,
and confidence intervals. The significance levels and
confidence intervals are computed by using degrees of freedom on
the t-distribution equal to N-s. Note that the estimates in
this analysis are the same as those in the first analysis, but
the estimated standard errors are slightly different.

The results from the mixed model analysis described in Section 3.3 are shown on page 14 of the SAS output. The significance levels are once again computed by using degrees of freedom equal to N-s. Note that the estimates as well as their estimated standard errors are slightly different than those given by the first two analyses.

The output on SAS pages 15-20 is obtained by using a model which allows for unequal carryover effects from the treatments occurring in the previous period. Page 17 gives an analysis appropriate when the H-F conditions are satisfied except for the test statistics and p-values which have been crossed out. Page 19 gives the analysis described in Section 3.2, and page 20 gives the analysis described in Section 3.3.

5. References


Appendix 1. SAS Analyses

The following statements were used to test for the H-F conditions for the experiment discussed in Section 4.

```sas
options ls=72 nodate pagesize=66;
dm 'log; clear; output; clear';
DATA one;
  INPUT seq subject y1 y2 y3 @@;
  IF seq=1 THEN trt1='A'; IF seq=1 THEN trt2='B'; IF seq=1 THEN trt3='C';
  IF seq=2 THEN trt1='A'; IF seq=2 THEN trt2='C'; IF seq=2 THEN trt3='B';
  IF seq=3 THEN trt1='B'; IF seq=3 THEN trt2='A'; IF seq=3 THEN trt3='C';
  IF seq=4 THEN trt1='B'; IF seq=4 THEN trt2='C'; IF seq=4 THEN trt3='A';
  IF seq=5 THEN trt1='C'; IF seq=5 THEN trt2='A'; IF seq=5 THEN trt3='B';
  IF seq=6 THEN trt1='C'; IF seq=6 THEN trt2='B'; IF seq=6 THEN trt3='A';
CARDS;
1 1 20.1 20.3 25.6 1 2 23.3 24.8 28.7 1 3 23.4 24.8 28.3
1 4 19.7 21.3 25.7 1 5 19.2 20.9 25.9 1 6 22.2 22.0 26.2
2 7 24.7 29.4 27.5 2 8 23.8 28.7 24.1 2 9 23.6 26.4 25.0
2 10 20.2 26.2 21.4 2 11 19.8 23.7 23.3 2 12 21.5 25.5 20.8
3 13 24.3 23.2 30.1 3 14 26.4 26.4 32.3 3 15 19.9 23.7 25.5
3 16 23.9 26.8 30.8 3 17 20.5 23.2 26.3 3 18 21.8 23.6 29.1
4 19 20.9 27.5 24.3 4 20 21.9 28.6 23.1 4 21 22.0 27.4 24.5
4 22 23.3 30.7 26.6 4 23 18.8 27.9 24.6 4 24 24.6 29.8 26.6
5 25 24.0 21.8 21.6 5 26 25.9 23.7 23.9 5 27 25.5 22.0 23.4
5 28 27.9 25.4 24.4 5 29 25.3 26.4 25.8 5 30 25.7 24.7 24.9
6 31 23.2 18.9 23.8 6 32 23.9 21.5 25.4 6 33 28.0 25.3 28.1
6 34 24.6 22.7 23.8 6 35 27.7 23.5 25.6 6 36 21.5 18.1 22.8
RUN;

PROC GLM DATA=one OUTSTAT=two;
  CLASS seq;
  MODEL y1--y3 = seq / NOUNI;
  REPEATED period 3 POLYNOMIAL / PRINTE;
RUN;

DATA sigma; SET two;
  IF _TYPE_ = 'ERROR';
  column1 = y1/df;
  column2 = y2/df;
  column3 = y3/df;
  KEEP column1--column3;
RUN;

DATA df; SET two;
  IF _TYPE_ = 'ERROR';
```
IF _NAME_ = 'Y1';
KEEP DF;
RUN;

DATA a; SET one; DROP y1-y3 trt1-trt3;
period=1; y=y1; trt=trt1; prior_trt='0'; OUTPUT;
period=2; y=y2; trt=trt2; prior_trt=trt1; OUTPUT;
period=3; y=y3; trt=trt3; prior_trt=trt2; OUTPUT;
RUN;

PROC PRINT DATA=a;

PROC GLM DATA=a;
CLASS seq subject trt period;
MODEL y = seq subject(seq) trt period;

ESTIMATE 'Trt A LSM' intercept 6 seq 1 1 1 1 1 1 trt 6 0 0
   period 2 2 2 /DIVISOR=6;
ESTIMATE 'Trt B LSM' intercept 6 seq 1 1 1 1 1 1 trt 0 6 0
   period 2 2 2 /DIVISOR=6;
ESTIMATE 'Trt C LSM' intercept 6 seq 1 1 1 1 1 1 trt 0 0 6
   period 2 2 2 /DIVISOR=6;

ESTIMATE 'Trt A-Trt B' trt 1 -1 0;
ESTIMATE 'Trt A-Trt C' trt 1 0 -1;
ESTIMATE 'Trt B-Trt C' trt 0 1 -1;

CONTRAST 'Trt A LSM' intercept 6 seq 1 1 1 1 1 1 trt 6 0 0
   period 2 2 2;
CONTRAST 'Trt B LSM' intercept 6 seq 1 1 1 1 1 1 trt 0 6 0
   period 2 2 2;
CONTRAST 'Trt C LSM' intercept 6 seq 1 1 1 1 1 1 trt 0 0 6
   period 2 2 2;

CONTRAST 'Trt A-Trt B' trt 1 -1 0;
CONTRAST 'Trt A-Trt C' trt 1 0 -1;
CONTRAST 'Trt B-Trt C' trt 0 1 -1;
RANDOM subject(seq);
RUN;

PROC SORT DATA=a;
   BY seq period;

PROC MEANS DATA=a NOPRINT;
   BY seq period; VAR y;
   OUTPUT OUT=b MEAN=ybar N=n;

DATA means; SET b;
   KEEP ybar;
RUN;
DATA size; SET b;
  IF period=1;
  k = 1/n;
  KEEP n k;
RUN;

  /* No Carryover Model */;
PROC IML;
RESET nolog;

USE sigma;
READ ALL INTO sigmahat; PRINT,,sigmahat;

USE df;
READ ALL INTO df;

USE means;
READ ALL INTO means;
muhat = means[,1];

USE size;
READ ALL INTO size;
n=size[,1];  k=size[,2];
d = DIAG(k);
sigmastr = d @ sigmahat;

H = { 1 1 0 0 0 0 0 1 0 0 1 0 0 ,
      1 1 0 0 0 0 0 0 1 0 0 0 1 0 ,
      1 1 0 0 0 0 0 0 0 1 0 0 1 0 0 ,
      1 0 1 0 0 0 0 1 0 0 1 0 0 0 1 ,
      1 0 1 0 0 0 0 0 0 1 0 1 0 0 0 ,
      1 0 1 0 0 0 0 0 0 1 0 0 0 1 0 ,
      1 0 0 0 1 0 0 0 0 1 0 0 0 0 1 ,
      1 0 0 1 0 0 0 0 0 1 0 0 0 0 0 ,
      1 0 0 1 0 0 0 0 0 0 0 0 0 1 0 ,
      1 0 0 1 0 0 0 0 0 0 0 0 1 0 0 ,
      1 0 0 0 0 1 0 0 0 1 0 0 0 0 1 ,
      1 0 0 0 1 0 0 0 0 0 0 1 0 0 0 ,
      1 0 0 0 0 1 0 0 0 0 0 1 0 0 0 ,
      1 0 0 0 0 0 1 0 0 0 0 1 0 0 0 ,
      1 0 0 0 0 0 0 1 0 0 0 0 1 0 0 ,
      1 0 0 0 0 0 0 0 1 0 0 0 0 1 0 ,
      1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 ,
      1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 ,
      1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 ,
      1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 ,
      1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 ,
      1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 };

PRINT / 'No Carryover Model';;
P R I N T
\'\'\'--------------------------------------------------------
  \'\'\'--
PRINT 'Multivariate Approach';
P R I N T
-'-----------------------------------------------'
-

a = { 6 1 1 1 1 1 6 0 0 2 2 2,
6 1 1 1 1 1 0 6 0 2 2 2,
6 1 1 1 1 1 0 0 6 2 2 2,
0 0 0 0 0 0 0 6 -6 0 0 0 0,
0 0 0 0 0 0 0 6 0 -6 0 0 0,
0 0 0 0 0 0 0 0 6 -6 0 0 0};/6;

DO I=1 TO 6;
  b = GINV(H')*a[i,];
estimate = b' * muhat;
  stderr = SQRT( b' * sigmastr * b);
  t = estimate/stderr;
  alpha=2*(1-PROBT(ABS(t),df));
  alpha = max(.0001,alpha);
  tcrit=tinv(.975,df);
  LCL=estimate-stderr*tcrit; UCL=estimate+stderr*tcrit;
  IF I=1 THEN;
    PRINT, 'Trt A LSM ' estimate stderr t alpha, 'A 95% CI is'
      LCL UCL;
    IF I=2 THEN;
      PRINT, 'Trt B LSM ' estimate stderr t alpha, 'A 95% CI is'
      LCL UCL;
    IF I=3 THEN;
      PRINT, 'Trt C LSM ' estimate stderr t alpha, 'A 95% CI is'
      LCL UCL;
    IF I=4 THEN;
      PRINT, 'Trt A-Trt B' estimate stderr t alpha, 'A 95% CI is'
      LCL UCL;
    IF I=5 THEN;
      PRINT, 'Trt A-Trt C' estimate stderr t alpha, 'A 95% CI is'
      LCL UCL;
    IF I=6 THEN;
      PRINT, 'Trt B-Trt C' estimate stderr t alpha, 'A 95% CI is'
      LCL UCL;
  END;

PRINT 'Mixed Model Approach';
P R I N T
-'-----------------------------------------------'
-

beta = GINV(H' * INV(sigamstr) * H) * H' * INV(sigamstr) *
muhat;
DO I=1 TO 6;
estimate = a[i,] * beta_eg;
stderr = SQRT( a[i,] * GINV( H' * INV(sigmastr) * H ) * a[i,]');
t = estimate/stderr;  alpha = 2 * (1 - PROBT(ABS(t),df));
alpha = max(.0001, alpha);
tcrit=tinv(.975,df);
LCL = estimate - stderr*tcrit;  UCL = estimate + stderr*tcrit;
IF I=1 THEN;
PRINT,, 'Trt A LSM ' estimate stderr t alpha, 'A 95% CI is' LCL UCL;
IF I=2 THEN;
PRINT,, 'Trt B LSM ' estimate stderr t alpha, 'A 95% CI is' LCL UCL;
IF I=3 THEN;
PRINT,, 'Trt C LSM ' estimate stderr t alpha, 'A 95% CI is' LCL UCL;
IF I=4 THEN;
PRINT,, 'Trt A-Trt B' estimate stderr t alpha, 'A 95% CI is' LCL UCL;
IF I=5 THEN;
PRINT,, 'Trt A-Trt C' estimate stderr t alpha, 'A 95% CI is' LCL UCL;
IF I=6 THEN;
PRINT,, 'Trt B-Trt C' estimate stderr t alpha, 'A 95% CI is' LCL UCL;
END;

/* Model with Carryover */;
PROC GLM DATA=a;
  CLASSES seq subject trt period priortrt;
  MODEL y = seq subject(seq) trt period priortrt/E;
  CONTRAST 'Carryover Effect' priortrt 1 -1 0 0,
      priortrt 1 0 -1 0;
  ESTIMATE 'Trt A LSM' intercept 18 seq 3 3 3 3 3 3 trt 18 0 0
      period 6 6 6 priortrt 4 4 4 6/DIVISOR=18;
  ESTIMATE 'Trt B LSM' intercept 18 seq 3 3 3 3 3 3 trt 0 18 0
      period 6 6 6 priortrt 4 4 4 6/DIVISOR=18;
  ESTIMATE 'Trt C LSM' intercept 18 seq 3 3 3 3 3 3 trt 0 0 18
      period 6 6 6 priortrt 4 4 4 6/DIVISOR=18;
  ESTIMATE 'Trt A-Trt B' trt 1 -1 0;
  ESTIMATE 'Trt A-Trt C' trt 1 0 -1;
  ESTIMATE 'Trt B-Trt C' trt 0 1 -1;
  ESTIMATE 'Carryover A-B' priortrt 1 -1 0 0;
  ESTIMATE 'Carryover A-C' priortrt 1 0 -1 0;
  ESTIMATE 'Carryover B-C' priortrt 0 1 -1 0;
CONTRAST 'Trt A LSM' intercept 18 seq 3 3 3 3 3 3 trt 18 0 0 period 6 6 6 priortrt 4 4 4 6;
CONTRAST 'Trt B LSM' intercept 18 seq 3 3 3 3 3 3 trt 0 18 0 period 6 6 6 priortrt 4 4 4 6;
CONTRAST 'Trt C LSM' intercept 18 seq 3 3 3 3 3 3 trt 0 0 18 period 6 6 6 priortrt 4 4 4 6;
CONTRAST 'Trt A-Trt B' trt 1 -1 0;
CONTRAST 'Trt A-Trt C' trt 1 0 -1;
CONTRAST 'Trt B-Trt C' trt 0 1 -1;
CONTRAST 'Carryover A-B' priortrt 1 -1 0 0;
CONTRAST 'Carryover A-C' priortrt 1 0 -1 0;
CONTRAST 'Carryover B-C' priortrt 0 1 -1 0;
RANDOM subject(seq);
RUN;

PROC IML;
RESET nolog;
USE sigma;
READ ALL INTO sigmahat;
USE df;
READ ALL INTO df;
USE means;
READ ALL INTO means;
muhat = means[,1];
USE size;
READ ALL INTO size;
n=size[,1]; k=size[,2];
d = DIAG(k);
sigamstr = d @ sigmahat;
H = { 1 1 0 0 0 0 0 1 0 0 1 0 0 0 0 1, 
      1 1 0 0 0 0 0 0 1 0 0 1 0 1 0 0, 
      1 1 0 0 0 0 0 0 0 1 0 0 1 0 0 0, 
      1 0 1 0 0 0 0 1 0 0 1 0 0 1 0 0, 
      1 0 1 0 0 0 0 0 1 0 1 0 1 0 0 0, 
      1 0 1 0 0 0 0 0 0 1 0 0 1 0 1 0, 
      1 0 0 1 0 0 0 0 0 1 0 1 0 1 0 0, 
      1 0 0 1 0 0 0 0 0 0 1 0 0 1 0 1, 
      1 0 0 1 0 0 0 0 0 0 0 1 0 1 0 0, 
      1 0 0 1 0 0 0 0 0 0 0 0 1 0 1 0, 
      1 0 0 1 0 0 0 0 0 0 0 0 0 1 0 1, 
      1 0 0 1 0 0 0 0 0 0 0 0 0 0 1 0, 
      1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1, 
      1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0, 
      1 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0, 
      1 0 0 0 1 0 0 0 0 0 0 1 0 0 0 1, 
      1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 1, 
      1 0 0 0 1 0 0 0 0 0 0 0 0 1 0 1, 
      1 0 0 0 1 0 0 0 0 0 0 0 0 0 1 1, 
      1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1, 
      1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0};
PRINT 'Model with Carryover';
P R I N T '------------------------------------------'
P R I N T '------------------------------------------'
PRINT 'Multivariate Approach';
P R I N T '------------------------------------------'
PRINT '------------------------------------------'

a = {18 3 3 3 3 3 3 18 0 0 6 6 6 4 4 4 6, 18 3 3 3 3 3 3 0 18 0 6 6 6 4 4 4 6, 18 3 3 3 3 3 3 0 0 18 6 6 6 4 4 4 6, 0 0 0 0 0 0 0 18 -18 0 0 0 0 0 0 0, 0 0 0 0 0 0 0 18 0 -18 0 0 0 0 0 0, 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0, 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 18 -18 0 0, 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 18 -18 0 0, 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 18 -18 0 0}/18;

DO I=1 TO 9;
  b = GINV(H')*a[i,]';
estimate = b' * muhat;
stderr = SQRT( b' * sigmastr * b);
t = estimate/stderr;
alpha = 2 * (1 - PROBT(ABS(t),df)); alpha = max(.0001, alpha);
tcrit=tinv(.975,df);
LCL = estimate - stderr*tcrit; UCL = estimate + stderr*tcrit;

IF I=1 THEN;
  PRINT,, 'Trt A LSM ' estimate stderr t alpha, 'A 95% CI is'
  LCL UCL;
IF I=2 THEN;
  PRINT,, 'Trt B LSM ' estimate stderr t alpha, 'A 95% CI is'
  LCL UCL;
IF I=3 THEN;
  PRINT,, 'Trt C LSM ' estimate stderr t alpha, 'A 95% CI is'
  LCL UCL;
IF I=4 THEN;
  PRINT,, 'Trt A-Trt B' estimate stderr t alpha, 'A 95% CI is'
  LCL UCL;
IF I=5 THEN;
  PRINT,, 'Trt A-Trt C' estimate stderr t alpha, 'A 95% CI is'
  LCL UCL;
IF I=6 THEN;
  PRINT,, 'Trt B-Trt C' estimate stderr t alpha, 'A 95% CI is'
  LCL UCL;
IF I=7 THEN;
  PRINT,, 'Carryover A - Carryover B ' estimate stderr t alpha;
IF I=8 THEN;
  PRINT,, 'Carryover A - Carryover C ' estimate stderr t alpha;
IF I=9 THEN;
   PRINT, 'Carryover B - Carryover C' estimate stderr t alpha;
END;

PRINT/'----------------------------------------------------------
PRINT 'Mixed Model Approach';
P '----------------------------------------------------------'

beta_eg = GINV(H' * INV(sigmas) * H) * H' * INV(sigmas) * muhat;

DO I=1 TO 9;
   estimate = a[i,] * beta_eg;
   stderr = SQRT( a[i,] * GINV( H' * INV(sigmas) * H ) * a[i,]');
   t = estimate/ stderr; alpha = 2 * (1 - PROBT(ABS(t),df) );
   alpha = max(.0001, alpha);
   tcrit=tinv(.975,df);
   LCL = estimate - stderr*tcrit; UCL = estimate + stderr*tcrit;
   IF I=1 THEN;
      PRINT, 'Trt A LSM ' estimate stderr t alpha, 'A 95% CI is'
   LCL UCL;
   IF I=2 THEN;
      PRINT, 'Trt B LSM ' estimate stderr t alpha, 'A 95% CI is'
   LCL UCL;
   IF I=3 THEN;
      PRINT, 'Trt C LSM ' estimate stderr t alpha, 'A 95% CI is'
   LCL UCL;
   IF I=4 THEN;
      PRINT, 'Trt A-Trt B' estimate stderr t alpha, 'A 95% CI is'
   LCL UCL;
   IF I=5 THEN;
      PRINT, 'Trt A-Trt C' estimate stderr t alpha, 'A 95% CI is'
   LCL UCL;
   IF I=6 THEN;
      PRINT, 'Trt B-Trt C' estimate stderr t alpha, 'A 95% CI is'
   LCL UCL;
   IF I=7 THEN;
      PRINT, 'Carryover A - Carryover B' estimate stderr t alpha;
   IF I=8 THEN;
      PRINT, 'Carryover A - Carryover C' estimate stderr t alpha;
   IF I=9 THEN;
      PRINT, 'Carryover B - Carryover C' estimate stderr t alpha;
END;
The important parts of the output obtained from the preceding SAS commands is shown next. All of the output can be obtained by executing the preceding commands.

### SAS

#### General Linear Models Procedure
Repeated Measures Analysis of Variance

Partial Correlation Coefficients from the Error SS&CP Matrix
of the Variables Defined by the Specified Transformation / \( \text{Prob} > |r| \)

<table>
<thead>
<tr>
<th>DF</th>
<th>PERIOD.1</th>
<th>PERIOD.2</th>
<th>PERIOD.1</th>
<th>PERIOD.2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.000000</td>
<td>0.050046</td>
<td>0.0</td>
<td>0.7892</td>
</tr>
<tr>
<td></td>
<td>0.7892</td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Test for Sphericity:** Mauchly's Criterion = 0.9910303

Chi-square Approximation = 0.2612936 with 2 df

\( \text{Prob} > \text{Chi-square} = 0.8775 \)

Manova Test Criteria and Exact F Statistics for
the Hypothesis of no PERIOD Effect
\( H = \) Type III SS&CP Matrix for PERIOD
\( E = \) Error SS&CP Matrix

\( S=1 \quad M=0 \quad N=13.5 \)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
<th>F</th>
<th>Num DF</th>
<th>Den DF</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wilks' Lambda</td>
<td>0.18884865</td>
<td>62.2811</td>
<td>2</td>
<td>29</td>
<td>0.0001</td>
</tr>
<tr>
<td>Pillai’s Trace</td>
<td>0.81115135</td>
<td>62.2811</td>
<td>2</td>
<td>29</td>
<td>0.0001</td>
</tr>
<tr>
<td>Hotelling-Lawley Trace</td>
<td>4.29524555</td>
<td>62.2811</td>
<td>2</td>
<td>29</td>
<td>0.0001</td>
</tr>
<tr>
<td>Roy’s Greatest Root</td>
<td>4.29524555</td>
<td>62.2811</td>
<td>2</td>
<td>29</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Manova Test Criteria and F Approximations for
the Hypothesis of no PERIOD*SEQ Effect
\( H = \) Type II SS&CP Matrix for PERIOD*SEQ
\( E = \) Error SS&CP Matrix

\( S=2 \quad M=1 \quad N=13.5 \)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
<th>F</th>
<th>Num DF</th>
<th>Den DF</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wilks’ Lambda</td>
<td>0.01748514</td>
<td>38.0625</td>
<td>10</td>
<td>58</td>
<td>0.0001</td>
</tr>
<tr>
<td>Pillai’s Trace</td>
<td>1.7330847</td>
<td>38.9581</td>
<td>10</td>
<td>60</td>
<td>0.0001</td>
</tr>
<tr>
<td>Hotelling-Lawley Trace</td>
<td>13.2652621</td>
<td>37.1427</td>
<td>10</td>
<td>56</td>
<td>0.0001</td>
</tr>
<tr>
<td>Roy’s Greatest Root</td>
<td>7.66493366</td>
<td>45.9896</td>
<td>5</td>
<td>30</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

**NOTE:** F Statistic for Roy’s Greatest Root is an upper bound.

**NOTE:** F Statistic for Wilks’ Lambda is exact.

### SAS

#### General Linear Models Procedure
Repeated Measures Analysis of Variance
Tests of Hypotheses for Between Subjects Effects

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type III SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEQ</td>
<td>5</td>
<td>53.1885</td>
<td>10.6377</td>
<td>1.04</td>
<td>0.4142</td>
</tr>
<tr>
<td>Error</td>
<td>30</td>
<td>307.7789</td>
<td>10.2593</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
General Linear Models Procedure
Repeated Measures Analysis of Variance
Univariate Tests of Hypotheses for Within Subject Effects

Source: PERIOD

<table>
<thead>
<tr>
<th>DF</th>
<th>Type III SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
<th>Adj Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>106.64518519</td>
<td>53.3259259</td>
<td>58.52</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Source: PERIOD*SEQ

<table>
<thead>
<tr>
<th>DF</th>
<th>Type III SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
<th>Adj Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>365.94037037</td>
<td>36.59403704</td>
<td>40.16</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Source: Error(PERIOD)

<table>
<thead>
<tr>
<th>DF</th>
<th>Type III SS</th>
<th>Mean Square</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>54.46777778</td>
<td>0.91112963</td>
<td></td>
</tr>
</tbody>
</table>

Greenhouse-Geisser Epsilon = 0.9911
Huynh-Feldt Epsilon = 1.2378

Applied Statistics in Agriculture
### Multivariate Approach

<table>
<thead>
<tr>
<th>ESTIMATE</th>
<th>STDERR</th>
<th>T</th>
<th>ALPHA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trt A LSM</td>
<td>23.655556</td>
<td>0.334641</td>
<td>70.726734</td>
</tr>
<tr>
<td></td>
<td>LCL</td>
<td>UCL</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>95% CI is 22.972489 24.338622</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESTIMATE</th>
<th>STDERR</th>
<th>T</th>
<th>ALPHA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trt B LSM</td>
<td>22.361111</td>
<td>0.334641</td>
<td>67.977726</td>
</tr>
<tr>
<td></td>
<td>LCL</td>
<td>UCL</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>95% CI is 22.053044 23.419178</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ESTIMATE</th>
<th>STDERR</th>
<th>T</th>
<th>ALPHA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trt C LSM</td>
<td>26.930556</td>
<td>0.334641</td>
<td>80.518516</td>
</tr>
<tr>
<td></td>
<td>LCL</td>
<td>UCL</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>95% CI is 26.247489 27.613622</td>
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<td></td>
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</table>

<table>
<thead>
<tr>
<th>ESTIMATE</th>
<th>STDERR</th>
<th>T</th>
<th>ALPHA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trt A-Trt B</td>
<td>0.9194444</td>
<td>0.2249851</td>
<td>4.0866897</td>
</tr>
<tr>
<td></td>
<td>LCL</td>
<td>UCL</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>95% CI is 0.4599635 1.3709254</td>
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</table>

<table>
<thead>
<tr>
<th>ESTIMATE</th>
<th>STDERR</th>
<th>T</th>
<th>ALPHA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trt A-Trt C</td>
<td>-3.275</td>
<td>0.2249851</td>
<td>-14.55652</td>
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<tr>
<td></td>
<td>LCL</td>
<td>UCL</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>95% CI is -3.734481 -2.815519</td>
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</table>

<table>
<thead>
<tr>
<th>ESTIMATE</th>
<th>STDERR</th>
<th>T</th>
<th>ALPHA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trt B-Trt C</td>
<td>-4.194444</td>
<td>0.2249851</td>
<td>-18.64321</td>
</tr>
<tr>
<td></td>
<td>LCL</td>
<td>UCL</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>95% CI is -4.653925 -3.734963</td>
<td></td>
<td></td>
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</tbody>
</table>

### Mixed Model Approach

<table>
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<tr>
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<th>STDERR</th>
<th>T</th>
<th>ALPHA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trt A LSM</td>
<td>23.666882</td>
<td>0.3342378</td>
<td>70.808514</td>
</tr>
<tr>
<td></td>
<td>LCL</td>
<td>UCL</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>95% CI is 22.984278 24.349487</td>
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</table>
### Applied Statistics in Agriculture

#### General Linear Models Procedure

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type III SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEQ</td>
<td>5</td>
<td>53.67258</td>
<td>10.73452</td>
<td>16.70</td>
<td>0.0001</td>
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<tr>
<td>SUBJECT (SEQ)</td>
<td>30</td>
<td>307.77889</td>
<td>10.25930</td>
<td>10.23</td>
<td>0.0001</td>
</tr>
<tr>
<td>TRT</td>
<td>2</td>
<td>249.72636</td>
<td>124.86318</td>
<td>124.41</td>
<td>0.0001</td>
</tr>
<tr>
<td>PERIOD</td>
<td>1</td>
<td>15.12500</td>
<td>15.12500</td>
<td>15.08</td>
<td>0.0002</td>
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#### ESTIMATE

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### Multivariate Approach

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### Model with Carryover

**SAS**

**Model with Carryover**

**Multivariate Approach**

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**Carryover**

**SAS**

**Model with Carryover**

**Mixed Model Approach**
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LCL and UCL 95% CI for ESTIMATE:
- Trt A: LCL = 22.988913, UCL = 24.376547
- Trt B: LCL = 22.155137, UCL = 23.542771
- Trt C: LCL = 26.096721, UCL = 27.484355
- Trt A-B: LCL = 0.3282782, UCL = 1.3392744
- Trt A-C: LCL = -3.613306, UCL = -2.60231
- Trt B-C: LCL = -4.447082, UCL = -3.436086

Carryover A-B: 0.285747, Carryover A-C: 0.4408652, Carryover B-C: 0.7266117