Educational Considerations

Volume 20 | Number 1 Article 7

9-1-1992

A Methodology to Determine the Sensitivity of Horizontal Equity Measures to Detect Changes in Distributional Equity

Sidney Castle

Recommended Citation

Castle, Sidney (1992) "A Methodology to Determine the Sensitivity of Horizontal Equity Measures to Detect Changes in Distributional Equity," *Educational Considerations*: Vol. 20: No. 1. https://doi.org/10.4148/0146-9282.1489

With the courts being presented with a new round of challenges to state public educational finance systems it is important, especially in view of continued poor state fiscal conditions, that researchers are able to more accurately identify the extent to which past and contemplated new school finance reform efforts affect educational resource distributional equity.

A Methodology to Determine the Sensitivity of Horizontal Equity Measures to Detect Changes in Distributional Equity

Sidney Castle

An awareness that a quality public education system was important to society, and concerns that marked differences in tax resources at the local district level adversely affected public education programs, culminated in the early 1970s in a series of legal challenges to state educational finance systems. The action of California's Supreme Court in overturning that state's public school funding plan became the ground breaking case (*Serrano v. Priest*, 1971) that led to litigation challenging educational finance systems in over half of the states.

Attempts by school finance theorists since the early 1920s to address inequities based on wealth have had as a common intent the provision of state funding in an inverse relationship to local district wealth so that relatively poorer districts would be compensated for their lack of resources (Jordan & McKeown,1980). The difficulty in attempting to codify school finance theory into state legislation and articulated educational resources allocation systems became apparent as courts questioned the extent to which state school finance systems addressed the issue of whether local school districts had equal ability in terms of revenue, to provide students with equal opportunities for learning (Webb, McCarthy, & Thomas, (1990).

The distributional equity definition of Friedman & Wiseman (1978), Bezeau's (1979) definition of utilizing Dalton's principles of transfers and Sen's Pigou–Dalton condition, and Bern & Stiefel's (1979) classification of equity concerns into two broad groupings with four conceptual groupings of horizontal equity measures, afford a descriptive base upon which school finance researchers have attempted to build statistical models to measure the degree of equity in state educational resource systems. Yet, as Garms (1978) has noted, there is no commonly agreed upon methodology for testing the extent to which school finance reforms have been effective.

A review of the literature shows that researchers increasingly tend to use multi-measure statistical models as opposed to the use of a single statistical measure when attempting to measure equity in state educational resource systems. Since Berne & Stiefel (1984) found that the measure used made a difference when assessing horizontal equity over time in the states of Michigan and New York, increased use of multi-measure statistical models affords a broader perspective from which to assess the effect over time of school reform.

The statistical models employed have enabled researchers to note the impact of school finance reform attempts in terms of general trends in distributional equity. Researchers like Jones & Salmon (1985) found that Virginia's school finance formula implemented in 1974–75 resulted in increased disparities in per-pupil revenues; other researchers like Cohn (1984) reported that in South Carolina school finance reform resulted in considerable improvements in equity. Berne & Stiefel (1983) reported finding evidence on the national level that horizontal equity improved from 1940 to 1960 and declined in the period between 1960 and 1977, while Odden (1986) in a summary of school finance reforms reported significant progress in the 1970s for increased equity issues in spite of the states' generally poor fiscal condition.

With the courts being presented with a new round of challenges to state public educational finance systems it is important, especially in view of continued poor state fiscal conditions, that researchers are able to more accurately identify the extent to which past and contemplated new school finance reform efforts affect educational resource distributional equity. This requires an understanding of how sensitive the most frequently used statistical measures are in detecting the amount of change that occurs over time in distributional equity of state educational resource systems. A difficulty arises from the fact that the most frequently used statistical measures tend to be non-parametric and differ significantly in the range of their computed values.

Study Methodology

Differences in the distributional equity of an educational resource system may occur between year-1 (Y_1) and beyond (Y_3) as the result of reform efforts. Computed values for two measures (A and B) used to assess the amount of equity on educational resource distributions for Y_1 and Y_3 may vary both in terms of their computed values ($A_{Yi} \neq B_{Y1}$ and $A_{Y3} \neq B_{Y3}$), and in terms of the amount of change of their computed values between Y_1 and Y_3 ($A_{Y3-1} \neq B_{Y3-1}$) Correlational analysis of the amount of change in computed values for statistical measures used to determine distributional equity and the amount of actual change noted in distributional equity between Y_1 and Y_3 provides a means of assessing the sensitivity of various measures to changes in distributional equity.

Key to assessment of the sensitivity of various measures is the identification of a methodology which can be used to determine the actual amount of distributional inequity. Lorenz

Sidney Castle is an Assistant Professor, Louisiana State University

(1905) contended that plotted sets of figures for cumulative percentages of resources–units and individuals in a population in which wealth is distributed equally will always give a straight line. In a distribution with inequity the curve will always begin and end in the same points as with an equal distribution, but will be bent in the middle. The extent of the bend, and therefore the area circumscribed by the curve, is said by Lorenz to be an indication of the extent of the equity in the distribution.

Berne & Stiefel (1984) applied the Lorenz curve to the study of horizontal equity and noted that when Lorenz's rule relative to X_1 and Y_1 values is observed, and every pupil receives the same object, then the Lorenz curve will be a 45° line running from the lower left to the upper right corners of the graph. Their definition of distributional characteristics resulting in a 45° for a plotted Lorenz curve is identical to their definition of horizontal equity also advanced by Jordan & McKeown (1980). Berne & Stiefel (1984)further noted that as long as there is some inequity in the distribution, the Lorenz curve will lie below the 45° line, and the greater the inequity the farther below the 45° line the Lorenz curve will lie.

Since the Lorenz curve is a plotted graph (Lorenz, 1905; Lows, 1984; Berne & Stiefel, 1984) which corresponds to the definition of horizontal equity, then the area of the curve can be mathematically described and computed in terms of a graph of *f* from a to b. The area circumscribed by the plotted sets of figures for an educational resource allocation distribution can be accurately measured by use of Simpson's Rule from calculus (Swokowski, 1984) and used as a benchmark by which to assess the sensitivity of horizontal equity measures to changes in distributional equity.

A database of fifty-four state-by-year distributions reported by three year intervals was used to assess the sensitivity of frequently used horizontal equity measures. The state-by-year distributions were derived from multiple year student and expenditure data sets received from state education agencies based on a stratified sample. The first strata represented states which had undergone court mandated restructure of their public educational resource allocation systems. It was presumed that multiple year data obtained from these states would reflect a reduction in distributional inequity. The second strata represented states which independent of court mandates had restructured their educational resources allocation systems and have been annually certified as being wealth-neutral by the Division of Impact Aid, U.S Department of Education. Because of the annual wealth-neutrality certification process it was assumed that multiple year data from states in the second strata represented little, or no, shifts in distributional equity.

The third strata represented states that reported an increasingly heavy reliance (i.e.≥ 85% of total funding) on local resources for public education funding. Multiple year data from states in the third strata were presumed to reflect high distributional inequity. All expenditures-per-pupil data were converted to constant dollars by the use of Consumer Price Index figures provided by the U.S. Bureau of the Census (1986) since Berne & Stiefel (1984) noted that some computed horizontal equity measures values changed as the result of inflation.

Three Simpson's Rule values were calculated with the Lorenz curve data sets for each of the state-by-year distributions of per-pupil expenditures. The three calculated Simpson's Rule values were next calculated between the three-year interval data sets from each state. Some of the state provided multiple three-year data sets which allowed additional comparisons to be made, (i.e. Y_3 vs. Y_1, Y_6 vs. Y_3, Y_6 vs. Y_1, etc.) so that a total of forty-three sets of comparison were obtained. Each of the horizontal equity measures examined in the study were calculated for the state-by-year distributions, and then amount of change values were calculated using the same multiple year data set format as used with Simpson's Rule values. Pearson r correlation coefficients were calculated using the forty-five comparison sets of Simpson's Rule and horizontal equity measure amount of change values. A computed Pearson r value of ≥ ± .50 was defined as constituting a meaningful correlation between Simpson's rule and horizontal equity measure relative amount of change values.

A total of twelve horizontal equity measures were examined in the study. All of the measures, except for Atkinson's index and the HGini coefficient, were described by Berne and Stiefel (1979) in their four conceptual groupings of horizontal equity measures. As Berne & Stiefel (1984) have noted, the Atkinson's Index is capable of focusing on different distribution. Changes in the assigned values for parameter E in the Atkinson's Index formula determine which part of the distribution will be emphasized (Atkinson, 1970). Parameter E values are set greater than zero and increases in the E value are said to correspond to a heavier emphasis on the lower part of the distribution. Table 1 identifies the twelve horizontal equity measures examined on the study and the seven different values for the E parameter of Atkinson's Index.

Table 1
Horizontal equity measures examined in the study

Name of Measure
Atkinson's Index (E = 0.5, 2, 8, 20, 40, 75, and 125)
Coefficient of Variation
Federal Range Ratio
Gini Coefficient
HGini coefficient (Hickrod formulation of Gini Coefficient)
McLoone Index
Range
Relative Mean Deviation
Restricted Range
Standard Deviation of Logarithms
Theil's measure
Variance

Study Results

A total of eighteen computed horizontal equity values (twelve different measures with seven different Atkinson's Index parameter E values) were compared to the three Simpson's Rule values using forty-three sets of amount of change data. Measures such as the Gini and HGini Coefficients were expected to correlate highly with the Simpson's Rule values since they are measures specifically formulated to approximate the area of the Lorenz curve. The McLoone Index is a measure that considers expenditure-per-pupil data for all students below the distribution median and was compared only to Simpson's Rule values for the lower-half of the Lorenz curve. Table 2 identifies those instances where computed Pearson r values met the established study criterion for meaningful correlation.

This study narrowly focused on the sensitivity of selected horizontal equity measures to detect changes in distributional equity over time. With respect to that narrow focus, the results indicated that:

(1) The Gini and HGini coefficients, formulated to approximate the area of the Lorenz curve, were found to be very sensitive to changes in distributional equity. The HGini coefficient displayed a marginally greater formulation with changes in the three areas of the distribution (total, upper-half and lower-half) than did the Gini coefficient. This may be explained by the formulation of the two measures. The Gini coefficient formula examines the relationship between all data sets in the distribution while the HGini coefficient formula focuses on the relationship of contiguous data sets in distribution. In instances of bimodal or multi-modal distribution configurations the Gini coefficient may tend to be more affected by distributional variance than the HGini coefficient.

https://newprairiepress.org/edconsiderations/vol20/iss1/7
DOI: 10.4148/0146-9282.1489

Table 2. Calculated Pearson r Values

Horizontal Equity Measure	Area of Lorenz Curve		
	Total	Upper	Lower
Atkinson's Index (E=.5)	–0.861	–0.892	–0.745
Atkinson's Index (E=2)	–0.927	–0.917	–0.873
Atkinson's Index (E=8)	–0.518	*	–0.555
Atkinson's Index (E=20)	*	*	*
Atkinson's Index (E=40)	*	*	*
Atkinson's Index (E=75)	*	*	*
Atkinson's Index (E=125)	*	*	*
Coefficient of Variation	0.545	0.580	*
Federal Range Ratio	0.727	0.712	0.696
Gini Coefficient	0.962	0.938	0.924
HGini Coefficient	0.972	0.943	0.941
McLoone Index	N/A	N/A	*
Range	*	*	*
Relative Mean Deviation	*	*	*
Restricted Range	0.507	0.555	*
Theil's measure	*	*	*
Std. Dev. of Logarithms	0.767	0.773	0.696
Variance	*	*	*

Note: * appears where Pearson r value < ± .500.

(2) Atkinson's Index was found to be sensitive to changes in distributional equity with low values of the parameter E (E= 0.5 to 8). This sensitivity was found to occur within a smaller range of E values than expected. The Atkinson's Index did display a slight shift in focus from the upper-half of the distribution (E= 0.5) to the total area of the distribution (E=8), however, the range of E values selected for this study were too narrow to allow a meaningful assessment of the relationship between E values and the sensitivity of the measure to detect changes in equity across different parts of the distribution.

(3) The Federal Range Ratio and Standard Deviation of Logarithms were found to be more sensitive to changes in distributional equity than either the Coefficient of Variation or the Restricted Range.

All four measures tend to be sensitive to all of the data points within a distribution but the Federal Range Ratio and Standard Deviation of Logarithms mitigate the impact of extreme outlier data points. The Federal Range Ratio and Standard Deviation of Logarithms are less sensitive to changes in distributional equity than are the Gini coefficient, the HGini coefficient or Atkinson's Index with E values of 0.5 to 8.

4) Measures such as the McLoone Index, Range, Relative Mean Deviation, Theil's measure, and the Variance did not meet the study criterion of Pearson r value of + .50. These measures either focus upon the deviation of single set of data from some central measure, or are descriptors of the variability in a distribution and are sensitive to extreme outlier data points.

Summary

A methodology was presented for assessing the sensitivity of horizontal equity measures to detect changes in distributional equity over time. The importance of being able to more accurately assess changes in distributional equity, and therefore the affect of school finance reform efforts, was discussed recognizing that the courts are again being presented with challenges to state school finance systems. A stratified sample of multiple state-by-year educational resource allocation distributions was used to assess the sensitivity of selected horizontal equity measures. The argument was raised for use of Simpson's Rule calculations of the three areas of Lorenz curve as a benchmark by which the selected horizontal equity measures were assessed. Simpson's Rule and horizontal equity measure values were first calculated using the state-by-year distributions. The amount of change in computed values between two state-by-year distributions were noted and forty-three sets of comparisons were obtained. Pearson r correlation coefficients were calculated using the comparison data sets. A study criterion of a computed Pearson r value of > + .50 was defined as acceptance of a horizontal equity measure's ability to detect changes in distributional equity over time. Measures which did meet the study criterion were identified and review was made of the formulation structure of the various measures selected for inclusion in the study.

References

Atkinson, A. B. (1970). On the measurement of inequality. *Journal of Economic Theory*, 245.

Berne, R., & Steifel, L. (1979). Taxpayer equity in school finance reform: The school finance and the public finance perspectives. *Journal of Education Finance, 5*, 109–132.

Berne, R., & Steifel, L. (1983). Changes in school finance equity: a national perspective. *Journal of Education Finance, 8*, 419–435.

Berne, R., & Steifel, L. (1984). *The measurement of equity in school finance.* Baltimore: John Hopkins University Press.

Bezeau, L. M. (1979). Measures of inequality of per pupil expenditure: Application to Ontario. *Journal of Education Finance, 5*, 133–148.

Cohn, E. (1984). Equity effects of the educational finance act in South Carolina. *Economics of Education Review, 3*, 333–339.

Garms, W. I. (1975). *Use of the Lorenz Curve and Gini index in school finance research.* Paper presented at the Annual Meeting of the American Education Research Association, 60th, Washington, D.C., March 31–April 4, 1975. (ERIC Document Reproduction Service No. ED 103 979).

Jones, H. B., & Salmon, R. G. (1985). A seven year fiscal equalization analysis of the Virginia public school finance program. *Journal of Education Finance, 10*, 524–534.

Jordan, K. F., & McKeown, M. P. (1980). Equity in financing public elementary and secondary schools. In J. Guthrie (Ed.), *School finance policies and practices.* (pp. 79–129). Cambridge MA: Ballinger

Lorenz, M. O. (1905). Methods of measuring the concentration of wealth. *Publications of the American Statistical Association, 9*, 209–219.

Odden, A. (1986). A school finance research agenda for an era of education reform. *Journal of Education Finance, 12*, 49–70.

Lows, R. L. (1984). Measurement of inequality: the Gini coefficient and school finance studies. *Journal of Education Finance, 10*, 83–94.

Serrano v. Priest, 487 P. 2d 1241 (Ca 1971).

Seodowski, E. W. (1984). *Calculus with analytic geometry* (3rd. ed.). Boston: Pindle, Weber & Schmidt.

U.S. Bureau of the Census (1986). *Statistical abstracts of the U.S.* (106th ed.). Washington, D. C.: Library of Congress Card No. 4–18089.

Webb, L. D., McCarthy, M. M., & Thomas, S. (1988). *Financing elementary and secondary education.* Columbus: Merrill.