Enhancements to the Discrete Generalized Multigroup Method

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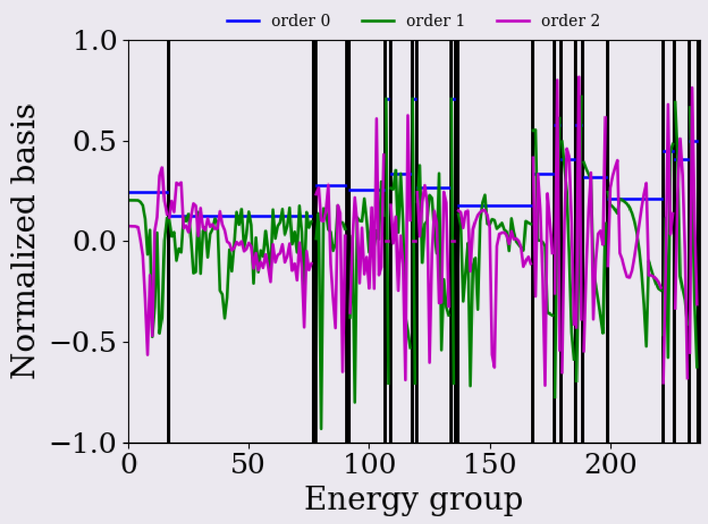
The Discrete Generalized Multigroup (DGM) Method

- The DGM method is a way to treat the energy variable in a numerical simulation [1].
- The method can work in conjunction with other schemes designed to treat the spatial and angular dependence.
- DGM uses an orthogonal basis to collapse the group structure while preserving much of the accuracy [2].
- The original method suffers from high memory costs since the angular flux must be stored [3].
- Present work explores several incremental improvements, which may provide a way to use DGM for systematic generation of broad-group cross sections with implicit, fine-group features

Basis Sets

The first three vectors within each coarse group for the 238-group structure for the POD basis using snapshots of UO_2 pin combined with MOX pin snapshots. Note that DGM requires that the zeroth order basis be the flat vector to decouple the higher order moments. The vectors are orthonormalized over each coarse group region. The 238 fine-groups have been collapsed to 23 coarse-groups.



The Discrete Generalized Multigroup (DGM) Equations

The DGM equations look similar to the multigroup form of the transport equations. In fact, the zeroth order equation is equivalent to the standard multigroup approximation. Note the presence of the δ term, which contains the angular dependence of the total cross section.

$$\hat{\Omega}\cdot\nabla\psi_{G,i}(\vec{r},\hat{\Omega})+\Sigma^t_{G,0}(\vec{r})\psi_{G,i}(\vec{r},\hat{\Omega})+\delta_{G,i}(\vec{r},\hat{\Omega})\psi_{G,0}(\vec{r},\hat{\Omega})$$
$$=\frac{1}{4\pi}\sum_{G'=1}^{N_G}\Sigma^s_{G\leftarrow G',i}(\vec{r})\phi_{G',0}(\vec{r})+\frac{\chi_{G,i}}{4\pi k}\sum_{G'=1}^{N_G}\nu\Sigma^f_{G'}(\vec{r})\phi_{G',0}(\vec{r})$$

where the coarse-group cross sections are defined as follows:

$$\Sigma^t_{G,0}(\vec{r})=\frac{\sum_{g\in G}\Sigma^t_g(\vec{r})\phi_g(\vec{r})}{\sum_{g\in G}\phi_g(\vec{r})}\qquad \nu\Sigma^f_G(\vec{r})=\frac{\sum_{g\in G}P^G_{ig}\nu\Sigma^f_g(\vec{r})\phi_g(\vec{r})}{\sum_{g\in G}\phi_g(\vec{r})}$$

$$\delta_{G,i}(\vec{r},\hat{\Omega})=\frac{\sum_{g\in G}P^G_{ig}\left(\Sigma^t_g(\vec{r})-\Sigma^t_{G,0}(\vec{r})\right)\psi_g(\vec{r},\hat{\Omega})}{\psi_{G,0}(\vec{r},\hat{\Omega})}$$

$$\Sigma^s_{G\leftarrow G',i}(\vec{r})=\frac{\sum_{g\in G}P^G_{ig}\sum_{g'\in G'}\Sigma^s_{g\leftarrow g'}(\vec{r})\phi_{g'}(\vec{r})}{\sum_{g'\in G}\phi_{g'}(\vec{r})}$$

$$\xi_{G,i}=\sum_{g\in G}P^G_{ig}\chi_g$$

Improvement 1 - Remove dependence on fine-group information

- We want an algorithm that only uses the fine-group data once
- To do so, redefine the cross sections in terms of flux moments to yield

$$\Sigma^t_{c,G,0}=\frac{\sum_{j=1}^{N_j}\phi_{c,G,0,j}\Sigma^{t*}_{c,G,0,j}}{\phi_{c,G,0,0}}\qquad \nu\Sigma^f_{c,G'}=\frac{\sum_{j=1}^{N_j}\phi_{c,G',0,j}\nu\Sigma^{f*}_{c,G',j}}{\phi_{c,G',0,0}}$$

$$\delta_{c,a,G,i}=\frac{\sum_{j=1}^{N_j}\psi_{c,a,G,j}\Sigma^{t*}_{c,G,i,j}}{\psi_{c,a,G,0}}-\Sigma^t_{c,G,0}\frac{\psi_{c,a,G,i}}{\psi_{c,a,G,0}}$$

$$\Sigma^s_{c,G\leftarrow G',l,i}=\frac{\sum_{j=1}^{N_j}\phi_{c,G',l,j}\Sigma^{s*}_{c,G\leftarrow G',l,i,j}}{\phi_{c,G,l,0}}$$

where

$$\Sigma^{t*}_{c,G,i,j}=\sum_{g=1}^{N^G_g}P^G_{ig}\Sigma^t_{c,g}P^G_{jg}$$

$$\nu\Sigma^{f,*}_{c,G',j}=\sum_{g'=1}^{N^{G'}_g}P^{G'}_{0g'}\nu\Sigma^f_{c,g'}P^{G'}_{jg'}$$

$$\Sigma^{s*}_{c,G\leftarrow G',l,i,j}=\sum_{g=1}^{N^G_g}P^G_{ig}\sum_{g'=1}^{N^{G'}_g}\Sigma^s_{c,g\leftarrow g',l}P^{G'}_{jg'}$$

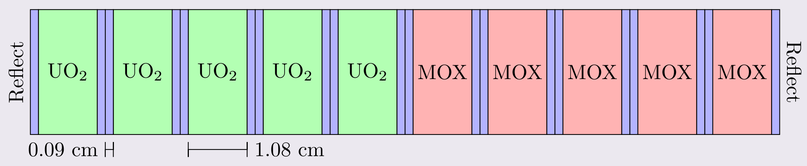
- These new definitions must still be recomputed every iteration
- The moment computations no longer use fine-group data

Old Algorithm

Input: cell and material properties, basis vectors
Compute χ and Q moments
Guess the initial, fine-group flux
while *not converged* **do**
  Compute flux moments
  Compute cross-section moments
  Solve zeroth-order equations ($i=0$)
  Update the eigenvalue
  for *all moments* $i>0$ **do**
    Solve ith-order equation
  end
  Reconstruct fine-group flux
end

The Test Problem

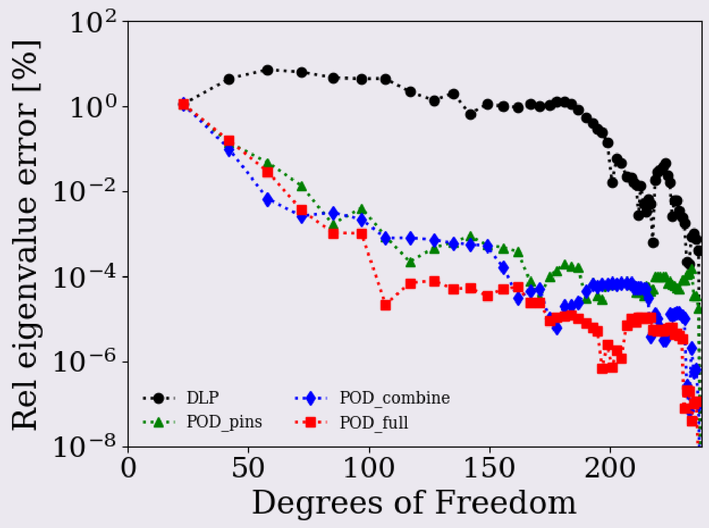
Depiction of the 10-pin problem. Each fuel section (UO_2/MOX) used 22 mesh cells, and each moderator section (blue) used 3 mesh cells.



The DGM method was implemented into a 1-D discrete ordinates code, which used the diamond difference approximation and 8 angles per half space.

Eigenvalue error for DGM method with truncation

Comparison of various basis sets used for the DGM method. Using DGM, a 1% error in the eigenvalue may be reached using approximately 58 degrees of freedom for a 238-group problem.



Note that at 238 degrees of freedom, all basis sets converge to within tolerance to the reference discrete ordinates solution (computed without DGM).

Improvement 2 - Approximate the angular flux

- The definition for $\delta_{G,i}(\vec{r},\hat{\Omega})$ depends on the angular flux
- This leads to a large memory footprint
- We can approximate the angular flux using a Legendre expansion

$$\psi_g(\vec{r},\hat{\Omega})=\sum_{l=0}^{\infty}P_l(\hat{\Omega})\phi_{g,l}(\vec{r})$$

- In this work, we consider both a zeroth and a first order expansion
- The definition for $\delta_{G,i}(\vec{r},\hat{\Omega})$ now becomes

$$\delta_{G,i}(\vec{r},\hat{\Omega})=\frac{\sum_{g\in G}P^G_{ig}\left(\Sigma^t_g(\vec{r})-\Sigma^t_{G,0}(\vec{r})\right)\sum_{l=0}^{N_l}P_l(\hat{\Omega})\phi_{g,l}(\vec{r})}{\sum_{l=0}^{N_l}P_l(\hat{\Omega})\sum_{g\in G}\phi_{g,l}(\vec{r})}$$

Improvement 3 - Spatially homogenize the cross sections

- The definitions for the cross section moments are functions of the flux
- Since the flux is spatially dependent, the moments are as well
- Thus, cross section data must be stored for each cell even if the cross sections are not normally spatially dependent
- We spatially homogenize the moments using flux weighted averaging
- The process is similar for all cross section moments
- Take for example the total cross section moments

$$\Sigma^t_{G,0}(R)=\frac{\sum_{c\in R}\Sigma^t_{G,0}(c)\Delta_c\phi_g(c)}{\sum_{c\in R}\Delta_c\phi_g(c)}$$

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New Algorithm

Input: cell and material properties, basis vectors
Compute χ and Q moments and expand fine-group cross-sections
Guess the initial, fine-group flux
Compute flux moments
while *not converged* **do**
  Compute coarse-group cross-section moments
  Solve zeroth-order equation ($i=0$)
  Update the eigenvalue
  for *all moments* $i>0$ **do**
    Solve ith-order equation
  end
end
Reconstruct fine-group flux

Improvement Cases

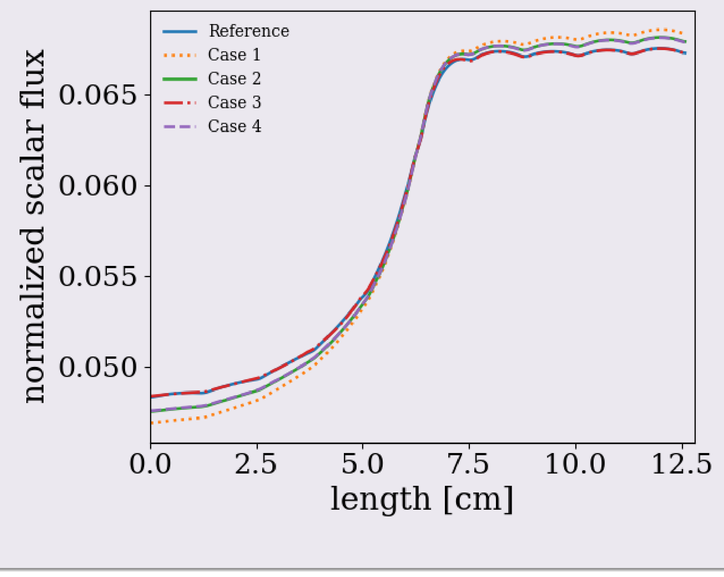
To evaluate the performance of the method, we define several cases

- Case 1: Assume flat angular flux (Improvement 2)
- Case 2: Assume linear angular flux (Improvement 2)
- Case 3: Spatial homogenization (Improvement 3)
- Case 4: Both linear angular flux and spatial homogenization

We also explored how these cases behave in both the full order problem and the truncated case.

Scalar flux for full problem for group 0

Fine-group scalar flux for the fastest energy group reconstructed using the DGM method. Improvement 3 causes slight deviation from the reference solution, whereas improvement 2 leads to a larger effect.



Case results for full problem compared to reference

The following table compares the DGM improvements to a discrete ordinates solution with no DGM. Note all data are relative percent error except for the first column, which are the reference values.

	Full-Ref	Full-(1)	Full-(2)	Full-(3)	Full-(4)
k_{eff}	1.122	-0.038%	-0.036%	0.040%	-0.021%
Cell 1	0.939	-0.471%	-0.288%	-0.096%	-0.310%
Cell 2	0.917	-0.456%	-0.279%	-0.099%	-0.303%
Cell 3	0.866	-0.425%	-0.261%	-0.104%	-0.286%
Cell 4	0.770	-0.380%	-0.237%	-0.107%	-0.256%
Cell 5	0.598	-0.335%	-0.218%	-0.088%	-0.204%
Cell 6	1.736	0.084%	-0.014%	0.280%	0.112%
Cell 7	1.151	0.447%	0.301%	-0.133%	0.193%
Cell 8	1.031	0.400%	0.277%	0.027%	0.268%
Cell 9	1.000	0.344%	0.241%	0.024%	0.235%
Cell 10	0.992	0.308%	0.218%	0.020%	0.210%

Case results for truncated problem compared to reference

The following table compares the DGM improvements to a discrete ordinates solution with no DGM. Note all data are relative percent error except for the first column, which are the reference values.

	Full-Ref	Trun.-Ref	Trun.-(1)	Trun.-(2)	Trun.-(3)	Trun.-(4)
k_{eff}	1.122	0.007%	-0.028%	-0.019%	0.014%	-0.011%
Cell 1	0.939	-0.078%	-0.612%	-0.418%	-0.125%	-0.435%
Cell 2	0.917	-0.091%	-0.601%	-0.416%	-0.143%	-0.436%
Cell 3	0.866	-0.126%	-0.586%	-0.419%	-0.183%	-0.442%
Cell 4	0.770	-0.203%	-0.586%	-0.450%	-0.266%	-0.473%
Cell 5	0.598	-0.405%	-0.678%	-0.586%	-0.456%	-0.590%
Cell 6	1.736	0.392%	0.459%	0.405%	0.526%	0.474%
Cell 7	1.151	0.004%	0.423%	0.280%	-0.033%	0.245%
Cell 8	1.031	-0.048%	0.382%	0.241%	-0.036%	0.241%
Cell 9	1.000	0.005%	0.407%	0.277%	0.015%	0.276%
Cell 10	0.992	0.025%	0.407%	0.285%	0.032%	0.280%

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