The Geometry Of Preposition Meanings

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ABSTRACT: This article presents a unified approach to the semantics of prepositions based on the theory of conceptual spaces. Following the themes of my recent book *The Geometry of Meaning*, I focus on the convexity of their meanings and on which semantic domains are expressed by prepositions. As regards convexity, using polar coordinates turns out to provide the most natural representation. In addition to the spatial domain, I argue that for many prepositions, the force domain is central. In contrast to many other analyses, I also defend the position that prepositions have a central meaning and that other meanings can be derived via a limited class of semantic transformations.

1. PROGRAM

The aim of this article is to present a unified approach to the semantics of prepositions based on the theory of conceptual spaces (Gärdenfors 2000, 2014). In most languages, prepositions form a closed class with a limited number of representatives. However, they are often used for a wide range of meanings. This semantic flexibility makes it difficult to provide an exhaustive analysis of their semantics. The linguistic literature on prepositions is extremely rich, and I have no ambition to do it justice. In this article, I focus on the convexity of their meanings and on which domains are expressed by prepositions. I argue that for many prepositions, the force domain is central. In contrast to many other analyses, I also defend the position that prepositions have a central meaning and that other meanings can be derived via a limited class of semantic transformations.

2. THE SEMANTICS OF PREPOSITIONS

Before I begin my semantic analysis I must present two technical concepts. The first is the notion of a domain (Gärdenfors & Löhndorf 2013; Gärdenfors 2014). Conceptual spaces (Gärdenfors 2000, 2014) are constructed out of quality dimensions. Examples are pitch, temperature, weight, size, and force. The primary role of the dimensions is to represent various “qualities” of objects in different domains. Some dimensions come in bundles — what I call domains — for example, space (which has dimensions of height, width, and depth); color (hue, saturation, and brightness); taste (salty, bitter, sweet and sour, and maybe a fifth dimension); emotion (arousal and value); and shape (where the dimensions are not well known).

The second is the notion of convexity. That a region R is convex means that for any two points x and y in R, all points between x and y are also in R. The motivation for applying betweenness to represent concepts in a conceptual space is that, if some objects located at x and y in relation to some domain are both examples of a concept, then any object that is located between x and y with respect to the same domain will also be an example of that concept.

Most prepositions can be grouped into two classes: locative, indicating where something is, and directional, indicating where something is going. Locative prepositions modify a noun (noun phrase) by specifying a location (a region) in the visuospatial domain.

The following thesis is the central claim of this article:

**Single-domain thesis for prepositions:** Prepositions represent convex sets of points or paths in a single domain.

As I will show, locative prepositions are represented by sets of points and directional prepositions by sets of paths.

The claim is that each use of a preposition builds on a single domain, but it is not required that all uses are based on the same domain. For example, I will argue that most typical uses of the prepositions “over,”
“on,” and “in” depend on the force domain. However, there are common metaphorical transformations of meanings that bring these prepositions into the visuospatial domain. As a matter of fact, metaphorical uses of prepositions are ubiquitous. Nevertheless, I will argue that for each preposition there is a central meaning that depends on a primary domain.

3. SPATIAL REPRESENTATION USING POLAR COORDINATES

3.1. Polar Coordinates and Convexity

To model the meaning of prepositions, I need to make some assumptions about how to model the visuospatial domain. Normally this domain is represented with the aid of the Cartesian coordinates \(x, y,\) and \(z,\) representing width, depth, and height, where distances are measured using a Euclidean metric. However, another way of representing space may be cognitively more realistic, namely, in terms of polar coordinates, which represent points in space in terms of distance and angles. Cognitively, a polar representation of space is more natural than a Cartesian one, since our visuospatial perceptual system is made for estimating directions and distances from ourselves rather than estimating distances between two points outside us. We are so influenced in our culture by Euclidean geometry, Cartesian coordinate systems, Newtonian mechanics, and Kantian a priori that we have difficulties seeing that there are other ways of describing spatial perception.

I start with a three-dimensional space \(S\) defined in terms of polar coordinates. It is assumed that the space has an origo point \(o.\) A point \(p\) is represented as a triple \(<r, \phi, \theta>\) where:

- \(r\) (the radius) is a real number (with \(r \geq 0\)) representing the distance of \(p\) from the origo;
- \(\phi\) (the azimuth angle) is the angle (with \(0^\circ \leq \phi < 360^\circ\)) between \(p\) and the “north” axis, perpendicular to the zenith (the azimuth);
- \(\theta\) (the polar angle) is the angle (with \(0^\circ \leq \theta \leq 180^\circ\)) between \(p\) and the “upward” axis (the zenith).

Following common practice in the use of polar coordinates, two absolute frames of reference are already built into the polar coordinates, namely, the zenith (up) and the azimuth (north), as the fixed reference directions relative to which other angles are defined. (In most cases, the upward direction is determined by gravitation.) Notice that when the polar angle is \(0^\circ\) or \(180^\circ\), then the value of \(\phi\) is arbitrary. We assume that the angle \(\phi\) goes clockwise when seen from above, so that east is \(90^\circ\) and west is \(270^\circ\).

Given the representation of polar coordinates, I can define a notion of polar betweenness that is different from the one generated by the standard Euclidean metric:

A point \(b = <x_b, \phi_b, \theta_b>\) lies polarly between a point \(a = <x_a, \phi_a, \theta_a>\) and a point \(c = <x_c, \phi_c, \theta_c>\) if there is some \(k, 0 < k < 1\) such that \(x_b = kx_a + (1-k)x_c, \phi_b = k\phi_a + (1-k)\phi_c,\) iff \(|\phi_a - \phi_c| \leq 180^\circ\), and \(\phi_b = k\phi_a + (1-k)(\phi_c - 360^\circ)\) iff \(|\phi_a - \phi_c| > 180^\circ\), and \(\theta_b = k\theta_a + (1-k)\theta_c\).
A region $R$ in $S$ is then defined to be polarly convex if and only if for all points $a$ and $b$ in $R$, any point $c$ that is polarly between $a$ and $b$ is also in $R$.

One thing to note about this definition, in comparison to Euclidean convexity, is under what coordinate transformations convexity is preserved. Euclidean convexity is preserved under multiplications, translations, and rotations of the coordinate system. In contrast, polar convexity only preserves convexity under multiplications (changing the values of the $r$-axis) and rotations (changing the values of the $\phi$- and $\theta$-axes). If a translation occurs, that is, if the origo moves, then convexity may not be preserved. It should also be noted that the only polarly convex set that allows both unrestricted multiplication and unrestricted rotation is the full space. The relevance of this will be apparent later.

In this context, Talmy (2000, chap. 1) proposes a topology principle that “applies to the meanings — or schemas — of closed-class forms referring to space, time, or certain other domains.” This principle, which he claims is a language universal, says that distance, size, shape, or angle from such schemas play no role for the meaning of these forms. He illustrates with the preposition “across,” which “prototypically represents motion along a path from one edge of a bounded plane to its opposite. But this schema is abstracted away from magnitude. Hence, the preposition can be used equally well in ‘The ant crawled across my palm,’ and in ‘The bus drove across the country.’ Apparently, no language has two different closed-class forms whose meanings differ only with respect to magnitude for this or any other spatial schema.”

### 3.2. Motion along Paths

The previous section introduced the formalism for representing the location of a trajector in terms of one vector. If a trajector is moving or if it is extended in shape, then the notion of a path is needed (see, e.g., Jackendoff 1983; Talmy 2000; Eschenbach et al. 2000; Zwarts 2005; and many others). There are different ways to represent a path, but I adopt the more common way of representing it as a directed curve, that is, as a continuous function $p$ from the real interval $[0, 1]$ to $S$. The values of the interval $[0, 1]$ do not represent moments of time, but they are an ordering mechanism. What is important is that the path represents “locations in sequence,” so to say.

The starting point of a path $p$ can be denoted as $p(0)$, the endpoint as $p(1)$, and for any $i \in (0, 1)$, $p(i)$ is an intermediate point. All of these will be points that are represented in terms of polar coordinates. It will be convenient later to refer to these coordinates in the following ways: $\text{radius}(p(i))$ is the radius of the path $p$ at $i$; $\text{polar}(p(i))$ is the polar angle of $p$ at $i$; $\text{azimuth}(p(i))$ is the azimuth angle of $p$ at $i$.

I also assume that the path is simple, that is, it does not cross itself. This can be defined by saying that for all $i$ and $j$, $p(i) \neq p(j)$.

### 4. Locative Prepositions

A preposition describes a relation between a trajectory and a landmark (“between” is an exception, since it involves a relation between a trajector and two landmarks). The landmark will usually not be point sized, but it will occupy an extended region of space. I assume a function $\text{loc}$ that assigns a subset $\text{loc}(x) \subseteq S$ as a region to every convex object $x$. For convenience, I often simply designate this region as $x$ and refer to it as “landmark.”

When we judge the relation between a trajectory and a landmark, the center of the landmark will function as the origo of $S$. To keep the mathematics from becoming too complicated, I consider only convex regions and restrict the analysis to circular landmarks with the origo in the center. It is an idealization that all landmarks have this shape, but for the meanings of the locative prepositions, this idealization does not result in any major deviations.

#### 4.1. Regions for Locative Prepositions

I now show that by using polar coordinates, locative prepositions can be given a highly systematic description that brings out more explicitly the spatial features of each preposition. A basic distinction in the system of prepositions is between internal and external regions, corresponding to the prepositions “inside” and “outside,” respectively. These regions can be defined as sets of points, where $r$ is the radius of the landmark. For simplicity’s sake, these regions are restricted to the horizontal plane by including only the horizontal angle in the coordinates.

inside: $\{<x, \phi>: x < r\}$
outside: \{<x, \phi>: x > r}\}
The corresponding regions can be diagrammed by shading the area
where the endpoints of the vectors are (see Figure 2).

The three coordinates of a polar system provide three ways of dividing
the space outside a landmark. The first is distance from a landmark (the
x-coordinate). A first distinction is to divide the space into points that
are near or far (see Figure 3). Of course, at what distance the division
is made depends on the context, in particular the size of the landmark:
what is near the sun in space covers a much larger distance than what
is near a golf ball on the ground. Moreover, the division is not sharp
but might allow for a vague gap between near and far.

The denotations of near and far can be given in the following way as
sets of located polar coordinates, for a given landmark region with ra-
dius r and a contextually given norm c for distance.
near: \{<x, \phi>: r < x < c\}
far: \{<x, \phi>: c < x\}

Working in the horizontal plane are prepositions like “in front of,” “be-
hind,” “beside,” “to the left of,” and “to the right of,” and the card-
inal directions “north of,” “west of,” “south of,” and “east of.” Much has
been written about the different frames of reference that are used here,
with different terminologies, such as egocentric and allocentric, rela-
tive and absolute, object-centered and viewer-centered (Levinson 1996;
Bohnemeyer 2012). I follow the terminology of Levinson. The cardinal
prepositions use an absolute frame of reference, directly tied to the fixed
(north) reference direction of \(\phi\). The other prepositions use either an
intrinsic frame of reference (based on features of the landmark itself)
or a relative frame of reference (based on the position of an observer).
I restrict the analysis to the intrinsic frame of reference by assuming
that some angle \(f \in \phi\) is assigned to a landmark \(x\) that represents the
front direction of \(\text{loc}(x)\). This results in a number of “focal” directions
from \(x\) in the horizontal plane, namely, \(f, b = f + 180^\circ, r = f + 90^\circ,\)
\(l = f - 90^\circ\). The regions for the prepositions can then be defined in
terms of closeness to these focal directions, leading to borders that are
vague and partially dependent on context:
in front of: \{<x, \phi>: r < x \text{ and } \phi \text{ is close to } f\}
behind: \{<x, \phi>: r < x \text{ and } \phi \text{ is close to } b\}
to the left of: \{<x, \phi>: r < x \text{ and } \phi \text{ is close to } l\}
to the right of: \{<x, \phi>: r < x \text{ and } \phi \text{ is close to } r\}

If the prepositions partition the horizontal angle into four regions,
then this would look as in Figure 4.
The preposition “beside” could be seen as the union of “left of” and “right of,” but it seems to have an extra element of proximity, which is lacking in the other horizontal prepositions. For the time being, I assume that “beside” covers angles of $\phi$ that are close to $\pm 90^\circ$, but I take a closer look at “beside” in the next subsection.

I next turn to the preposition “between,” which is, in its most prominent use, based on two landmarks (Habel 1989; Van der Zee & Watson 2004). In this case, the reference angle is determined by the line between the two landmarks. Each landmark can be seen as generating a cone going in the direction of the other landmark. The region representing “between” can then be defined as the set of points that belong to both cones.

The following definitions give us such regions for a landmark with radius $r$, given the appropriate notion of closeness:
above: \( \{<x, \phi, \theta>: x > r, \theta \text{ is close to } 0^\circ\} \)
below: \( \{<x, \phi, \theta>: x > r, \theta \text{ is close to } 180^\circ\} \)

In summary, dividing the space into regions along the three polar coordinates generates the regions for most of the common locative prepositions in a natural way, by imposing simple conditions on the magnitude of coordinates.

4.2. Convexity of Locative Prepositions

Given the notion of polar betweenness, the question is now whether regions of locative prepositions are polarly convex. The answer is clearly affirmative for the regions of the “angular” prepositions in the horizontal plane, namely, the ones that have one single “cone” or halfspace: “in front of,” “behind,” “to the left of,” “to the right of,” “north of,” “above,” “below,” and so on. They are all polarly convex according to the definition. Since intersections of convex regions are also convex, “between” satisfies the convexity principle. The reasoning is as follows. If the “cones” of the two landmarks are already convex, then the region that we create by intersecting these two regions (given the appropriate notion of intersection for regions of polar coordinates) will not create any discontinuities that violate convexity. It can be noted that the regions for all these prepositions are all closed under multiplication along the \(r\)-axis, but not under rotation.

Next consider “outside,” “near,” and “far,” which prima facie seem problematic for convexity, because there is a gap in the center of the region, where the landmark is, for all three prepositions, and, for “far,” the area that is near the landmark. If position \(a\) to the east of my house is outside (near, far) and a position \(b\) to the west is outside (near, far), then there are definitely positions in between that may not be outside (near, far). But this description depends on Euclidean betweenness. If polar betweenness is applied to these regions, then they are convex. The curved nature of the line between two points \(a\) and \(b\) leads around the gap in the middle. Hence a point in the middle of \(a\) to the east and \(b\) to the west is a point either to the north or to the south. This is illustrated in Figure 8.

Similarly, “inside” is straightforward, given the idealizing assumption about landmarks I have made. Note also that, in contrast to the previous class of prepositions, the regions for “outside,” “inside,” “near,” and “far” are all closed under rotations, but not under multiplication.

I am left then with the convexity of the preposition “beside.” A natural description is that the value of the angle \(\phi\) of the vector going from the landmark does not deviate considerably from either the left (-90°) or the right (+90°) direction of \(y\). In Figure 5, it seems that this set of vectors consists of two separate sets, which would then violate the convexity requirement. One conclusion might be that “beside” is in fact a nonconvex preposition, maybe because it has a disjunctive definition: vectors that are close to +90° or -90°. While intersections of convex regions are convex, unions are not necessarily so. However, another conclusion might be that I have misanalysed “beside.”

Are there ways to analyze it as convex? One possibility would be to say that “beside” covers all the horizontal directions, but its use for the forward and backward direction is preempted or blocked by the prepositions “in front of” and “behind.” Another possibility would be to restrict “beside” to just one side at a time, as if we are saying “at a side.”

In fact, there are not many complex prepositional phrases that violate convexity, only some artificial cases like “diagonally above” and “exactly one or exactly two meters above.” Even run-of-the-mill modified cases like “two feet above” or “far outside” are convex (given our assumptions about landmarks). Not only basic prepositions are con-
vex, but even many of the complex ones. This means that convexity gains additional support as a general semantic constraint for locational prepositions. Of course, the principle should also be analyzed cross-linguistically, but that is beyond the scope of this article.

5. DIRECTIONAL PREPOSITIONS

I now consider prepositions that are used to express how a trajector moves relative to a landmark, which are the subject of much study (a recent example is Pantcheva 2010). They include the following:

- **Goal prepositions**: to, into, onto, toward
- **Source prepositions**: from, out of, off, away from
- **Route prepositions**: through, over, along, around, across

In addition, it is possible to get directional readings for locative prepositions:

- **Goal**: (to go) under, behind, ...
- **Source**: (to come) from under, from behind, ...
- **Route**: (to pass) under, behind, ...

The source needs to be marked by the source preposition “from,” while the possibility of a goal or route interpretation depends very much on the verb and other factors (see, e.g., Gehrke 2008; Nikitina 2008).

5.1. Representing Directional Prepositions as Sets of Paths

Most of the directional prepositions can be represented as imposing a locative condition on a particular part of the path, for instance, on the endpoint (goal) or starting point (source). Stricter and weaker definitions are possible, as discussed in Zwarts (2005), but here I use definitions that only involve opposite conditions on the starting point and endpoint:

**Goal prepositions**

to = \{p: \text{near}(p(1)) \text{ and not near}(p(0))\}
to = \{p: \text{inside}(p(1)) \text{ and not inside}(p(0))\}
(to) behind = \{p: \text{behind}(p(1)) \text{ and not behind}(p(0))\}

**Source prepositions**

from = \{p: \text{near}(p(0)) \text{ and not near}(p(1))\}

5.2. Convexity of Directional Prepositions

It is not obvious how to define betweenness for path. If I focus on simple paths that are defined as mappings from the interval [0, 1] to

out of = \{p: \text{inside}(p(0)) \text{ and not inside}(p(1))\}
from behind = \{p: \text{behind}(p(0)) \text{ and not behind}(p(1))\}

**Route prepositions**

by, past = \{p: \text{not near}(p(0)), \text{not near}(p(1)), \text{and there is an } i \in (0,1) \text{ such that } \text{near}(p(i))\}
through = \{p: \text{not inside}(p(0)), \text{not inside}(p(1)), \text{and there is an } i \in (0,1) \text{ such that } \text{inside}(p(i))\}
(via) behind = \{p: \text{not behind}(p(0)), \text{not behind}(p(1)), \text{and there is an } i \in (0,1) \text{ such that } \text{behind}(p(i))\}

Other prepositions compare the endpoint with the starting point:

-toward = \{p: \text{radius}(p(1)) < \text{radius}(p(0)) \text{ and for all } i \in [0, 1] \text{ } \text{radius}(p(i)) > r\}
away from = \{p: \text{radius}(p(0)) < \text{radius}(p(1)) \text{ and for all } i \in [0, 1] \text{ } \text{radius}(p(i)) > r\}

I next come to a class of directional prepositions that are at a level of greater complexity than the others, namely, “around,” “across,” and “along.” Notice that these are also morphologically complex, derived from expressions of shape or orientation like “round,” “cross,” “long.” What is different from the other route prepositions is that they are specified not in terms of a location at a particular point of the path but in terms of the shape or orientation of the path as a whole. *Around* paths are round in some sense, *across* paths are orthogonal to the main axis of the landmark, *along* paths parallel.

I have to set aside “across” and “along,” because they involve landmarks that are elongated, which goes beyond my simple model of the landmark as a region around an origin. This leaves “around,” which is a preposition with a wide range of meanings (Zwarts, 2003). A very strict interpretation of “around” (restricted to the horizontal plane) takes it as corresponding to a full and perfect circular path:

around = \{p: \text{there is an } r > 0 \text{ and an } \alpha \in (0^\circ, 360^\circ), \text{ such that } p(i) = (r, \alpha \pm i \phi)\}

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points \( <x, \phi> \) in the horizontal surface, there is a solution that follows the standard definition: Let \( p_1(i) \) and \( p_2(i) \), where \( 0 \leq i \leq 1 \), be two functions mapping onto \( <x_1(i), \phi_1(i)> \) and \( <x_2(i), \phi_2(i)> \) respectively. Then the path \( p_3(i) \) is said to be *polarly between* \( p_1(i) \) and \( p_2(i) \), if and only if there is some \( k, 0 \leq k \leq 1 \), such that for all \( i, 0 \leq i \leq 1 \), \( p_3(i) = <x_3(i), \phi_3(i)> = k<x_1(i), \phi_1(i)> + (1 - k)<x_2(i), \phi_2(i)> \).

Given these definitions, it is easy to prove that the meanings of the goal prepositions are convex. Convexity for “into” and “out of” follows from the convexity of the points in the landmark, which I have assumed to be convex. Similarly, the convexity of “to” follows from the fact that the region for “near” is convex. To be precise, if \( p_1(i) \) and \( p_2(i) \) are two paths such that \( \text{near}(p_1(1)) \) and \( \text{near}(p_2(1)) \), then for any path \( p_3 \) between \( p_1 \) and \( p_2 \), it will also hold that \( \text{near}(p_3(1)) \). An analogous argument holds for “from,” and similar arguments can be made for “(to) behind” and “from behind,” since the region for “behind” is convex.

The convexity of “toward” is easy to prove: if \( p_1(i) \) and \( p_2(i) \) are two paths such that \( \text{radius}(p_1(1)) < \text{radius}(p_1(0)) \) and for all \( i \in [0, 1] \) \( \text{radius}(p_1(i)) > r \), and \( \text{radius}(p_2(1)) < \text{radius}(p_2(0)) \) and for all \( i \in [0, 1] \) \( \text{radius}(p_2(i)) > r \), then for any path \( p_3 \) between \( p_1 \) and \( p_2 \), it will also hold that \( \text{radius}(p_1(1)) < \text{radius}(p_3(0)) \) and for all \( i \in [0, 1] \) \( \text{radius}(p_3(i)) > r \). An analogous argument shows that “away from” is convex.

Our way to handle route prepositions is to view them as sequences of two conjoined paths: *by* or *past* a landmark means that the first path goes *near* the landmark, and the second path goes *away* from the landmark; *through* a landmark means that the first path goes *into* the landmark, and the second path goes *out of* the landmark; and going *behind* a landmark means that the first path goes *to* *behind* the landmark, and the second path goes *from* *behind* the landmark. Given these descriptions, the convexity of the route prepositions follows essentially from the convexity of the set of points that conjoin the two paths, by analogy with the convexity for the goal prepositions. A point conjoining two paths \( p \) and \( q \) is a point for which \( p(1) = q(0) \). It is interesting to note that with the definition of betweenness for paths, conjoining paths preserves convexity: If \( P \) and \( Q \) are convex sets of paths and \( P \diamond Q \) is the set of conjoinings of paths from \( P \) and \( Q \), then \( P \diamond Q \) is convex too.

I finally turn to the convexity of “around.” The corresponding sets of paths are not polarly convex. Take the “around” paths \( p_1 \) and \( p_2 \) that have the same radius \( r = 1 \) and the same starting angle \( \alpha = 0° \) but go in opposite directions (clockwise versus counterclockwise). Consider now a path \( p_3 \) that also has a radius \( r = 1 \) and an angle that is always exactly halfway between the angles of \( p_1 \) and \( p_2 \). This path is not an around path. In fact, it is not even a proper path because it maps only to the angles 0° and 180°.

A way out of this problem is to consider the *direction* of paths, so that the relation of betweenness is restricted to paths that have the same direction. If we make this restriction, polar betweenness can also be defined for “around,” and it can then be shown to be convex.

6. DOMAINS OF PREPOSITIONS

The second part of the single-domain thesis about prepositions concerns their domains. The traditional semantic approach to prepositions is that they express *spatial* relations. For example, Leibniz (1765, chap. 3, §1) writes that prepositions “are all taken from space, distance and movement, and then transferred to all sorts of changes, orders, sequences, differences.” This *localist* view has then been a main trend in linguistics (Miller & Johnson-Laird 1976; Jackendoff 1983; Landau & Jackendoff 1993; Herskovits 1986; Zwarts & Winter 2000). When combined with nonspatial words, they create a spatially structured mental representation of the expression. For example, Herskovits (1986) presents an elaborate study of the fundamental spatial meanings of prepositions, and she argues that the spatial structure is transferred by metaphorical transformations to other contexts.

However, although the localist program has been successful for most locative and directional prepositions, recent analyses of prepositions have indicated that domains other than the visuospatial domain may be central for the meaning of some prepositions. First of all, there exist prepositions that refer to the *temporal* domain. In English, the clearest examples are “before” and “after.” (The convexity of their temporal regions is immediate.) Etymologically, these words have a spatial origin, but they are now used primarily for the temporal domain. The spatial meanings have been taken over by “in front of” and “behind.”

How can it be ascertained that the proper domain for “before” and “after” is the time dimension and not a spatial dimension? First note
that all four prepositions are invariant under multiplication of the underlying dimension. There is, however, an asymmetry between “after” and “behind” in the following examples:

(1) She is behind me in the queue, but if I turn around she is in front of me.

(2) *She is after me in the queue, but if I turn around she is before me.

These examples show that a reversal of spatial orientation changes the preposition “behind” to its opposite “in front of.” However, a spatial reversal does not change the temporal ordering from “before” to “after.” This means that the meanings of “before” and “after” allow some spatial rotations, which is evidence that the prepositions are not based on the spatial domain.

As I will show, this argument concerning rotations can be generalized.

7. A FORCE-DYNAMIC ANALYSIS OF IN, ON, AND AGAINST

Several authors have proposed that the meanings of many prepositions include a force-dynamic component (Dewell 1994; Bowerman 1996; Garrod et al. 1999; Tyler & Evans 2001; Zwarts 2010; Bélien 2002).

I begin by considering the domains of “in” and “on.”

I will use force vectors to analyze the dynamic aspects of these prepositions, although I cannot work out this idea in as much detail as the locative and directional prepositions, because the domain of forces is much more complex. Mathematically, what is needed here is the notion of a force field, that is, a space with a force vector associated with each point of that space. In most practical situations, such a force field would almost always involve gravitation as a component (this force has direction $\theta = 180^\circ$). However, force fields may be extremely complex depending on the relations between the landmark and the trajectory, and it is therefore difficult to give a general analysis based on them.

Herskovits (1986) noted that the pear in Figure 9a is considered to be in the bowl even though it is not spatially inside the bowl. If the apples are removed, but the pear is left in exactly the same spatial position as in Figure 9b, then the pear is no longer in the bowl. So spatial location is not sufficient to determine whether an object is in a

bowl. In Figure 9a, the reason why the pear is in the bowl is that it is physically supported by the apples, while in 9b it has no such support. The notion of “support” clearly involves forces.

![Figure 9](from Garrod et al. 1999, p. 168)

Vandeloise (1986, pp. 222–224) analyzes the French dans (in) in terms of containment, which he describes as a functional relation related to the notion of “carrier” (porteur) in that the container “controls” the position of the contained, but not conversely (p. 229). Similarly, the position of Garrod et al. (1999) is that if x is in y, then y’s location “controls” x in the sense that the container y “constrains” the location of x (p. 173). They do not, however, specify what is meant by “constrains.” Also Zwarts (2010, p. 209) writes that in involves a passive and stative configuration of forces, not necessarily involving contact.

I propose that containment can be expressed as a counterfactual constraint based on forces: if the trajector were perturbed, it would still be controlled by forces exerted by the landmark. This condition clearly separates the situation in Figure 9a from that of 9b. I submit that this constraint captures the basic meaning of “in,” and consequently the central meaning of the preposition is based on the force domain. In most cases, this can be replaced by a simpler condition: if the landmark moves, so does the trajector. Clearly this is not the whole story about “in” and containment because it does not distinguish it from “on” and support. Coventry, Carmichael, and Garrod (1994) and Feist and Gentner (1998) show that the concavity of the landmark plays a role.
Subjects prefer “in” for dishes and “on” for plates.

As mentioned earlier, a general analysis of convexity in terms of the forces involved in the meaning of “in” is extremely complicated. However, for situations of objects in containers, such as apples in a bowl, the force patterns involved would, in general, support the convexity of “in.” I will not give a more detailed analysis.

Furthermore, because the forces controlling the figure have a spatial location, it is difficult to totally disentangle the force domain from the visuospatial domain and verify that only the force domain determines the meaning of “in.” To be sure, there are examples where “in” is used purely spatially without any forces involved as in (3) and (4). One could view these as metonymic extensions of the central meaning of “in.” In many of these cases, both “in” and the purely spatial “inside” can be used (e.g., “in the box,” “inside the box”). There are, however, cases where only “inside” can be used:

(3) The airplane is in the cloud.
(4) Oscar is in the middle of the room.
(5) inside the border, inside the city limits, inside the door
(6) *in the border, *in the city limits, *in the door

In (5) “inside” seems to take a boundary as its landmark and refer to a region at one of the two sides of the boundary. In contrast to “in,” then, “inside” is much more like the “axial” prepositions such as “behind” and “above” and hence clearly a spatial preposition.

Another example is illustrated in Figure 10. In Figure 10a, the movements of the duck are controlled by the movements of the tube, and according to the proposed criterion, it is appropriate to describe the situation as that the duck is in the ring. In contrast, Figure 10b shows a situation where the movements of the duck are less constrained by those of the tube, and consequently it is less appropriate to say that the duck is in the tube. It is more felicitous to say that the duck is inside or within the tube, since “inside” and “within” are prepositions that refer to the spatial domain.

To measure the container’s degree of control on the trajectory $x$, we can define the function $in(x)$ by saying that $in(x) = d$ if $d$ is the average distance that the container is perturbed before $x$ changes position. In Figure 10a, $in(duck) = 0$, since the duck moves as soon as the tube is perturbed. However, in Figure 10b, $in(duck)$ is the average distance of the duck from the sides of the tube, which is greater than zero.

Similarly, in Figure 9a, $in(pear) = 0$, but $in(pear) = \infty$ in Figure 9b, if the container is imagined to move only in the horizontal dimensions.

For the preposition “on,” the semantic representation involves contact and support from below. A spatial region is not sufficient to determine the meaning of “on.” In brief, I propose that the meaning of “$x$ is on $y$” is that the force vector from $x$ makes $x$ come in contact with $y$, and a counterforce from $y$ balances the force vector of $x$. Typically, the force vector pointing from the figure $x$ is generated by gravitation. (Just as with “in,” the meaning of “on” also involves counterfactual control: $x$ is on the table means that if the table were to move, so would $x$.)

How can it be established that the basic domain of “on” is the force domain? This is a problem because, in typical cases, the role of forces is not noted when “on” is applied. The reason is that the gravitational force is like the drone of bagpipe music: it is always there and normally not attended to. However, in other situations, the force dynamics are more transparent. Figure 11 illustrates a situation where the spatial relation involved in typical uses of “on” can be contrasted with the force-dynamic meaning of the preposition. The lamp is vertically above the balloon and in contact with it, which are the normal spatial conditions...
proposed for the meaning of “on.” Nevertheless it is odd to say that the lamp is on the balloon. As a matter of fact, it might be more natural to say that the balloon is on the lamp or against the lamp, since the lifting force from the balloon makes it come into contact with the lamp (if the lamp moves, so does the balloon, but not conversely). In this example, the directions of the forces involved are the opposite of what is typical.

The same principle applies to, for example, “the painting is on the wall,” “the Band-Aid is on the leg,” and “the button is on the shirt.” What is crucial in examples of this type is that there is a force that acts on the trajector that makes it remain in contact with the landmark, not the spatial direction of the trajectory in relation to the landmark. In the case of the painting on the wall, it is still gravitation that acts on the trajector, but now the hanging mechanism makes the painting exert a pressure in the direction of the wall. These examples support the claim that only the force domain is necessary to determine the meaning of “on.” More generally, this can be expressed by saying that “on” is invariant under spatial rotation as long as the force relations stay the same. As we have seen before, this provides evidence that the meaning of “on” depends on the force domain and not the spatial domain.\(^8\)

Regarding the convexity of “on,” the situation for the horizontal dimensions is relatively clear: if \(x\) and \(y\) are on an extended landmark \(u\) that is itself convex, and \(z\) is between \(x\) and \(y\), then \(z\) is also on \(u\), under the assumption that the force field operating on \(x\) and \(y\) toward \(u\) is convex, so that it also operates on \(z\). However, there is also a “vertical” sense of convexity (“vertical” in relation to the direction of the force). For example, if a book is on a table and a plate is on the book, then one can say that the plate is also on the table. This form of transitivity is limited to horizontally extended objects, though: if an apple is on a table and the stem is on the apple, then we do not infer that the stem is on the table. Nevertheless, if \(x\) is on \(u\), and \(y\), which is above \(x\), is also on \(u\), then any object between \(x\) and \(y\) is also on \(u\). In brief, “on” satisfies the convexity requirement if the relevant landmarks and force field have the required convexity properties.

Some authors speak of a functional analysis of prepositions rather than a force-dynamic one (Vandelooise 1986; Coventry et al. 1994; Garrod et al. 1999; Coventry et al. 2001; McIntyre 2007). I suggest that most of the functions used in these analyses can be reduced to forces or force patterns. For example, Garrod et al. (1999, pp. 173–174) defines the notion of a landmark \(y\) functionally supporting a trajector \(x\) as follows: “\(y\)’s location controls the location of \(x\) with respect to a unidirectional force (by default gravity) by virtue of some degree of contact between \(x\) and \(y\).” They next say that if \(x\) is on \(y\), then \(y\) functionally supports \(x\). In terms of forces, this can be expressed by saying that gravitation (or some other force) presses \(x\) toward \(y\), and the friction between \(x\) and \(y\) makes \(x\) move whenever \(y\) moves. Their analysis is congruent with the one presented here, though they do not explicitly mention the force domain and its geometric structure as a separate domain.

“Against” is perhaps the clearest example of a preposition that is based on the force domain (Zwarts 2010):

\[(7)\quad \text{Oscar bumped against the wall.}\]

Zwarts (2010, p. 194) notes that “against” “combines with verbs like crash, lean, push, bang, and rest, verbs that all involve forces.” In the typical case, such as (7), the trajector follows a more or less horizontal
path and exerts a force on a landmark. There are, however, also static uses of "against" where the path is reduced to a point (endpoint focus transformation):

(8) The ladder leans against the house.

Furthermore, in this case too the direction can be changed by a rotation:

(9) Standing on the wooden stairs, Oscar pressed his shoulders against the cellar flap, but he could not open it.

In (9) the direction of the force is vertical. Again, this suggests that "against" is invariant to rotational spatial transformations.

Bowerman and her colleagues (Bowerman & Pedersen 1992; Bowerman 1996; Bowerman & Choi 2001) analyze *aan*, *op*, and *in* in Dutch, as well as the Korean prepositional verbs *nehta* (put loosely, in or around) and *kkita* (fit tightly in). The analysis involves force components that cannot be reduced to spatial relations. Bowerman and Choi (2001) present a semantic map containing five steps that is divided into different areas by different prepositions in different languages.

8. OVER AS A FORCE RELATION

Within the tradition of cognitive semantics, the preposition "over" has been studied over and over again, beginning with Brugman (1981), then expanded by Lakoff ((1987) and partly reanalyzed by, for example, Vandeloise (1986), Dewell (1994), Kreitzer (1997), Tyler and Evans (2001), Zlatev (2003), and Belien (2008).

Coventry et al. (2001) propose that what distinguishes "over/under" from the purely spatial "above/below" is that "over/under" has a functional meaning. They showed subjects, for example, pictures of people wearing umbrellas protecting them with more or less success against the rain. Their results suggest that the use of "above/below" is determined by spatial relations, while "over/under" is sensitive to functional relations, for example, whether the umbrella is protecting a person from rain falling in a slanted direction. I agree that Coventry et al. (2001) are on the right track, but instead I would argue that "over" has a central meaning that is based on a relation in the force domain (see also Coseriu 2003; Van der Gucht et al. 2007; Belien 2008).

A common assumption in the kinds of lexical analyses performed in cognitive linguistics is that a word or an expression has a prototypical meaning, which can then be extended by different transformations. Brugman's (1981) and Lakoff's (1987) central image schema for "over" is depicted in Figure 12. The content of the schema can be formulated entirely in terms of spatial dimensions as the trajectory (TR) moving horizontally in a position vertically higher than a landmark (LM). A prototypical example is the following:

(10) The bird flies over the yard.

On this account, over typically describes a kinematic scene (in contrast to above which is stative).

![Figure 12: The central image schema for over according to Lakoff (1987, p. 426).](image)

Starting from this central schema, Lakoff then identifies twenty-four different senses of “over” that are connected to each other in radial network. Most of these senses can be described as elaborations (Holmqvist 1993) or superimpositions, where the different elements of the schema are further specified. I do not count such elaborations as alternative meanings of “over”; they are just specializations of other meanings. Tyler and Evans (2001) are more systematic, and they present criteria for when two meanings of a word are different. Nevertheless they end up with fourteen different meanings of “over.” Most of these can be generated from the central schema, just as in the case of Lakoff’s meanings, by metaphorical and metonymical transformations.

Instead I want to argue in favor of a minimal specification of the meaning of “over.” The crucial point concerning minimal specification versus full specification is that the semantics of a word must be separated from its role in a construal underlying a particular composite expression. Here I follow Fauconnier (1990, p. 400), who writes: ‘The ‘semantics’ of a language expression is the set of constraints it imposes
on cognitive constructions; this is a structural property, which is independent of context” (for a similar position, see Coseriu 2003). When several words are put together in a phrase or sentence, their constraints are combined in the cognitively most efficient manner. If the constraints are incompatible, a metaphorical or metonymical transformation is required to close the meaning gap. The different kinds of combinations of constraints will result in a variety of construals that may give the impression that a particular word is polysemous.

In contrast to most previous analyses, I therefore maintain that there is a single central meaning of “over” from which the other meanings are generated by various combinations and transformations. Dewell (1994) argues that the central meaning of “over” should be described as the trajector taking a semicircular path in relation to the landmark, as in Figure 13.

![Figure 13: The central image schema for over according to Dewell (1994, p. 353).](image)

The semicircular structure includes the feature that the trajector moves up and down along a path. In typical cases, the vertical dimension is associated with a gravitational force. Although Dewell only marginally considers the force relations, adding gravitation to the vertical axis entails that moving up along this dimension involves a force that is strong enough to overcome gravitation. Therefore, I propose that the central meaning of “over” is the schema in Figure 13, but including a force-dynamic element in the description of the path. In many cases, the countergravitational force is exerted by the trajector itself (e.g., “the bird flies over the yard”). Just as for “on,” the basic domain for “over” is therefore the force domain. In contrast to “on,” the central meaning of “over” involves a path and no contact between trajector and landmark.

For “in” and “on,” it is also assumed that the force of the trajectory is balanced by a counterforce from the landmark, but for “over” the forces of the landmark do not seem to play a role. Then, since force dynamics naturally lead to changes in spatial position, “over” generates a lot of implicatures for the visuospatial domain, but still, the force dynamics are primary.

Regarding the convexity of the meaning of “over,” this again depends on how betweenness relations for force vectors and paths are defined. I will not go into the details of an analysis.

A similar force-dynamic analysis of Dutch over is presented by Belien (2008). She describes the central constraint as follows: “Over designates a relation between a trajector and a landmark in which the trajector is related to the landmark by a mental path that follows a surface of the landmark, and from which a force points to the landmark” (p. 49). The central component here is that “over” involves a force that is directed toward the landmark. The situation is parallel to that of “on” and “in.” Just as for “on” and “in,” the primacy of the force domain can be exhibited by considering spatial rotations of the basic scheme.

In the schema in Figure 13, the canonical direction for “over” is the vertical dimension, determined by gravitation. In some cases, one finds transformations of the canonical direction:

(11) Victoria wears a veil over her face.
(12) Victoria held her hands over her eyes.

In both (11) and (12), the landmark is the face, and its canonical direction is transformed into the vertical dimension. In example (11), it is still gravitation that acts on the trajector, but now the fastening of the veil makes it exert a pressure in the horizontal direction toward the face. And in example (12), it is Victoria herself who exerts a force of her hand in the horizontal direction. From this perspective on the forces involved, the veil and the hands are over the face.

Other force directions can be involved as well:

(13) The fly is crawling over the ceiling.
(14) Oscar nailed a board over the hole in the ceiling.
In this example, the nailing creates an upward force that makes the trajector (the board) be directed toward the landmark (the ceiling). So in examples (13) and (14) the prevalent force is directed vertically upward, which is yet another example of invariance under spatial rotation (further examples are found in Coventry et al. 2001). The upshot is that the invariance of “over” under visuospatial rotational transformations strongly suggests that the force domain is primary for the meaning of “over.”

I next give some examples of the transformations involved in adapting the central meaning of “over” (Figure 13) to different construals. Dewell (1994, p. 355) points out that Lakoff’s schema (Figure 12) can be seen as a special case of his where the central region is profiled. This example brings up a first type of semantic transformation: profiling a segment of a schema (this can be seen as a special case of the metonymy operation). For example, in “The bird flew over the yard,” the central part of the bird’s flying path is attended to. In contrast, in “Sam fell over the cliff,” the downward part of the trajector is put in focus; and in “The plane climbed high over the city,” the focus is on the upward part of the trajector (p. 356). The profiling is not arbitrary, however; Dewell argues that a constraint for “over” is that the profiled segments must include the peak point of the arc (p. 355).

A special case of a profiling transformation is endpoint focus. In “[the city of] Sausalito is over the bridge” (Lakoff, 1987, p. 424), Sausalito is not moving vertically above the Golden Gate Bridge, but the speaker’s inner gaze makes a “fictive motion” (Talmy 1996) from San Francisco over the bridge to Sausalito. Focusing on the endpoint is a pars pro toto metonymy. This case can be seen as a counterexample to Dewell’s constraint that the peak point must be included. Admittedly, it engages another type of transformation.

The central meaning of “over” both involves a path (located vertically higher than the landmark) and requires that the trajector not be in contact with the landmark. In English, both these components can be modified by transformations:

(15) The car drives over the bridge (contact).
(16) The painting hangs over the fireplace (no path).

However, the two transformations cannot be combined. For example, a painting that is over the fireplace but in contact with it will be said to be on the fireplace, since the scene then fulfills the requirements for the basic meaning of “on.” Dewell (1994, p. 373) notes that when “over” involves contact, it is distinguished from “on” by involving a path (which can be a result of mental scanning) and by the trajector (or its path) covering the landmark (as in (15)).

Endpoint focus can be applied not only to spatial phenomena but also to processes. In “The play is over” (Lakoff, 1987, p. 439), the play is construed as an extended event that creates a path in time. If the end of the path is focused, the play has gone over this path. Thus this example has the same structure as “Sausalito is over the bridge,” except that the relevant domain for the endpoint focus is time, not space. Note that the use of “over” in “The play is over” is, consequently, metaphorical since the horizontal dimension is changed to the time dimension. “Over again” involves repeating an event and traversing the path repeatedly.

Finally, a few words about “under.” In many respects, this preposition behaves like a complement to “over,” though the force-dynamic features of “under” are different in some situations (see also Coventry et al., 2001). In contrast to “below,” which has its meaning in the spatial domain, “under” indicates some dynamic interaction between trajector and landmark. McIntyre (2007, p. 2) gives the following example:

(17) I washed it under/*below the shower.

However, I will not pursue the details of the analysis.

9. CONCLUSION

The main thesis of this article is that prepositions represent convex sets of points or paths in a single domain. By using polar coordinates to define betweenness, I have shown that both locational and directional prepositions in general fulfill the convexity criterion.

Regarding the thesis about a single domain, my claim is that many prepositions, traditionally believed to express spatial relations, involve force dynamics in their central meanings. A still unresolved question is whether the core meanings of, for example, “on,” “in,” and “over” can be expressed in the force domain only or their basic semantics also requires the visuospatial domain. For example, “over” and “against”
both involve paths in their core meanings, but this is a path that is associated with a force field.

One preposition that I have not considered is “at.” It is related to the spatial “by” and “toward,” but examples indicate that a goal domain is included in the meaning of “at.” Herskovits (1986) mentions that sitting at a desk or washing at a sink involves more than just being close to the desk or the sink. In these cases, there is an intentional component in being at a place. Similarly, Landau and Jackendoff (1993, p. 231) point out that throwing a ball toward someone is different from throwing a ball at someone. In the latter case, the throwing has an intention of hitting.

With this caveat concerning “at,” my tentative conclusion is therefore that locative and directional prepositions are based on either the visuospatial domain, the time domain, or the force domain. What complicates matters is that there are many metaphorical uses of prepositions.

Most important, the ordinary locative prepositions can be used in other domains, for example, “the temperature is above 20,” “Midsummer is behind us,” “The color of our neighbors’ house is near that of ours,” “Victoria is working toward her goals.” And, of course, prepositions based on the force domain can be used metaphorically for the visuospatial domain. Since the force vectors are also spatially located, this kind of metaphor is difficult to detect.

Since prepositions, in most languages, form a closed class, new meanings cannot be introduced easily by an additional word. So when an expression for a relation within a domain is required, in particular for domains different from the visuospatial domain or the force domain, the most similar prepositional meaning will have to be selected (cf. Tyler & Evans, 2001, p. 761). Often a metaphorical or metonymical transformation helps in closing the meaning gap. The context of the expression adds information too (cf. Tyler & Evans, 2001, p. 762).

The main topic of this article has been the use of geometric notions to describe the semantics of prepositions. Locative and directional prepositions are prime examples of how meaning is geometrically structured. I have also shown that locative and directional prepositions support the general idea that concepts can be represented by convex regions in the visuospatial domain. In contrast, the force domain is involved in the meaning of prepositions such as “in,” “on,” “over,” “under,” and “against.” Since the force fields involved depend to a large extent on the nature of the trajector and the landmark and their relation, it is difficult to determine to what extent the meaning of these prepositions represents convex regions in the force domain or some product space. This topic merits further investigation.

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Notes

1 This article builds on material from Chapter 11 of my book The Geometry of Meaning: Semantics Based on Conceptual Spaces (Gärdenfors 2014). A more elaborate version of the material in sections 3–6 is can be found in Zwarts and Gardenfors (submitted). I wish to thank Joost Zwarts for his collaboration on many of these topics. He has drawn several of the pictures.

2 In Gardenfors (2014), I present similar single-domain hypotheses also for adjectives and verbs.

3 See Gardenfors (2000, Section 3.5) for further details.

4 Coventry et al. (1994) suggest that beside involves a functional component, because subjects judge beside as more appropriate when trajector and landmark are functionally related, for example, a jug and a glass. If this is the case, then a spatial analysis is not sufficient.

5 Since before and after are invariant under multiplication, then if the prepositions were based on the spatial domain and they were invariant under rotation, their corresponding regions would be the full space, as argued earlier.

6 However, “inside the ring” is ambiguous, also meaning inside the tube itself.

7 It would be interesting to investigate whether the distinction between the Korean prepositions kkita (fitting tightly in) and netha (loose inclusion) (Bowerman & Choi 2001) can be expressed as the difference between whether in(x) = 0 or in(x) > 0.

8 This example indicates that an analysis of the invariance classes of different preposition is a strong tool for determining the relevant domains of prepositions.

9 This analysis contrasts with that of Lakoff (1987), who analyzes example (14) as a separate meaning of over, generated by a rotational transformation of the central meaning.
References


