SIMPLE STEP-STRESS TESTING WITH COVARIATE IN AGRICULTURE

Larry Cleair

Imad H. Khamis

Mohamad AI-Haj Ebrahem

Simple Step-Stress Testing with Covariate in Agriculture

Larry Cleair Southeast Missouri State University
Dr. Imad H. Khamis Southeast Missouri State University
Mohamad Al-Haj Ebrahem Kansas State University

Abstract

In industry product testing can be an expensive and time-consuming process. Testing design changes in long-lived products could cause lengthy delays in product introduction or improvement. As an alternative, accelerated life testing can quickly yield information on product life by exposing the product to conditions beyond those of normal design stress. To further streamline this process a two step-stress test will take all elements to failure in a relatively short time. Variables within the sample other than the one that we are controlling in the step-stress testing are uncontrolled but observed and are called covariates. A statistical relationship between the mean lifetime of the test unit and the covariate will allow a prediction of mean lifetime based on the covariate.

In agriculture, animals, or plants may be the test items and dosage of a chemical, amount of fertilizer, temperature, etc may be the stress variable. The breed of the animal or the variety of the plant may be the covariate. In this paper we suggest several potential applications of step-stress testing in agriculture and present inferential procedures for observations that are distributed exponentially.

1. Introduction

The use of accelerated life testing in industry and agriculture for product testing and development is a time and money saving device to test a product's life in a shorter period of time by shortening that life by exposing the product to a stress beyond its normal design limit. In simple step-stress testing that stress is increased at a predetermined time, τ , with the test run at the second stress level until all test units fail. Step-stress testing not only reduces the time required for testing but also eliminates the need for censoring since the test runs until all the subjects reach failure. Caution must be used in any accelerated life testing that the increased stress does not so drastically change the situation that different systems are affected. For example if we were testing the cool weather tolerance of wheat varieties and our stress levels kept the ground frozen we would not be testing what we set out to test. The efficiency and validity are improved if the time of the stress increase is optimized. Variables within the sample other than the one being controlled in the step-stress testing, such as: manufacturer, day of the week produced, or shelf-life, that are observed but uncontrolled are covariates. A statistical model that relates the covariate and the time to failure in lower or higher stress conditions will allow a life prediction based on the covariate. In this study the effect of the covariate was kept small to simulate the type of differences that would be expected if a product was produced to the same specifications at different plants or by different manufacturers. The interaction of the covariate with the stress increase at τ has up to this point never been tested for significance or estimated.

2. Examples in Agriculture

Example 1. The drought tolerance of two different breeds of soybeans could be tested by growing samples of each under conditions where they received 20% less water than their normal tolerance. Some plants may stop growing while others survive. The failure time of those plants that stop growing is recorded, and after a predetermined time the water is reduced to 40% less than normal, testing would end when all the plants stop growing. The covariate would be the two different breeds of soybeans that are tested.

Example 2. CO-OP and Heston brands of baler twine claim to have the same tensile strength. Samples of both are stressed at 200% of design stress. Some may fail while others survive the failure time is recorded, after a predetermined time the stress is increased to 400% of design stress and held there until all of the samples fail. The covariate is the brands of twine that are tested.

Example 3. To test the effect of crowding during confinement feeding on two breeds of chickens that have equal size and growth rates, samples of each may be put into conditions where the space per bird is 25% less than minimal and any birds that die or show less than profitable growth are removed and the time recorded and the space reduced to keep the space per bird constant. After a predetermined time the remaining birds are put into conditions where there space per bird is 50% less than normal until all the birds die or fail to grow at a profitable rate. The variety of chicken will be the covariate in this test.

3. A Mathematical Model for Simple Step-Stress Testing with Covariate

Procedure for Data Collection

All n test units, n_1 from covariate group 1 and n_2 from covariate group 2 where $n_1 + n_2 = n$ are initially placed at low stress and run until time τ when the stress is increased to high stress where the testing is continued until all units fail. We assume that a random number of units n_{ij} from covariate group j fail at stress $x_i, i = 1,2$ and time to failure t_{ijk}, $k = 1,2,...n_{ij}$, are observed and recorded on these test units.

Cumulative Exposure Model

The model used is termed the cumulative exposure model by Nelson, [1]. The exponential distribution or constant failure rate model will be used to parameterize the data. Let t denote the lifetime of a test unit of interest. The lifetime, t is said to have an exponential distribution if the cumulative distribution function is given by

$$F(t) = 1 - e^{(-\frac{t}{\theta})}, \text{where } t > 0,\ \theta > 0 .$$

In applications where the exponential is assumed to model the underlying distribution it is reasonable to assume that the parameter θ is a function of the stress. The change from design stress in step-stress testing will be reflected in a change in θ for each new stress level. To analyze data from a step-stress test, one needs a model that relates the life distribution under step-stressing to the distribution under constant stress. The relationship between θ and stress is given by

$$\log(\theta_i) = \beta_0 + \beta_1 x_i \quad (1)$$

for simple step-stress tests, by

$$\log(\theta_{ij}) = \beta_0 + \beta_1 x_i + \beta_2 w_j \quad (2)$$

for simple step-stress tests with covariates with no interaction between the covariate and the stress level, and

$$\log(\theta_{ij}) = \beta_0 + \beta_1 x_i + \beta_2 w_j + \beta_3 x_i w_j \quad (3)$$

for simple step-stress tests with covariates with interaction between the covariate and the stress level. In this paper we concentrate on model (3) showing that the interaction between the covariate group and the stress level is statistically significant.

The cumulative exposure model assumes that, at any time, the remaining life of a unit depends only on the exposure it has experienced, and not on how that exposure was accumulated. In other words large groups that were brought to the same survivors to originals ratio would have the same life distribution for their surviving members if exposed to the same stress no matter how that ratio was achieved.

For simple step-stress testing the cdf of the time to failure is given by Bia, Kim, Lee, [2]

$$G(t)=\begin{cases}1-e^{-\frac{t}{\theta_{1j}}} & 0<t<\tau \\ 1-e^{-\frac{t-\tau}{\theta_{2j}}-\frac{\tau}{\theta_{1j}}} & \tau<t<\infty\end{cases} \quad j=0,1 \text{ for groups } 1\,\&\,2 \text{ respectively}$$

where s_1 is the solution of $G_2(s_1)=G_1(\tau)$.

The basic assumptions of the model are:

(1) Testing is done at stress levels x_1 and x_2 where $x_0 \le x_1 < x_2$, x_0 being the design stress.

(2) The life distribution of a test unit is the exponential model.

(3) The scale parameter θ_{ij} at stress x_i and covariant group w_j is given by

$$\log(\theta_{ij})=\beta_0+\beta_1 x_i+\beta_2 w_j+\beta_3 x_i w_j .$$

(4) The lifetimes of the test units are independent and identically distributed.

(5) All n units are initially placed on low stress x_1 and run until τ when the stress is increased to x_2 with testing continuing at x_2 until all units fail, τ is the same for both covariate groups.

(6) β_0, β_1, β_2, and β_3 are unknown constants: independent of time and stress, and estimated from the test data.

(7) $n_{i0}+n_{i1}=n$ where n_{i0}, n_{i1} are the number of units in each covariant group and i is the stress level where the unit failed.

Under the assumption of an exponential distribution for G_i, the likelihood function from observation t_{ijk}, i=1,2, , k=1,2...,n_{ij} (j=0,1;I=1,0 for group 1 and group 2 respectively) is:

$$L(\beta_o,\beta_1,\beta_2,\beta_3)=\left[\prod_{k=1}^{n1}\frac{1}{\theta_{1j}}e^{\left(\frac{t_{1jk}}{\theta_{1j}}\right)}\prod_{k=1}^{n2}\frac{1}{\theta_{2j}}e^{\left(\frac{t_{2jk}-\tau}{\theta_{2j}}-\frac{\tau}{\theta_{1j}}\right)}\right]^{I}\left[\prod_{k=1}^{n1}\frac{1}{\theta_{1j}}e^{\left(\frac{t_{1jk}}{\theta_{1j}}\right)}\prod_{k=1}^{n2}\frac{1}{\theta_{2j}}e^{\left(\frac{t_{2jk}-\tau}{\theta_{2j}}-\frac{\tau}{\theta_{1j}}\right)}\right]^{(1-I)} .$$

After simplifications we obtain:

$$\begin{aligned}\log L(\beta_0,\beta_1,\beta_2,\beta_3) = {} & -n\beta_0-(n_{10}x_1+n_{20}x_2+n_{11}x_1+n_{21}x_2)\beta_1- \\ & (n_{10}w_0+n_{20}w_0+n_{11}w_1+n_{21}w_1)\beta_2- \\ & (n_{10}x_1w_0+n_{20}x_2w_0+n_{11}x_1w_1+n_{21}x_2w_1)\beta_3+ \\ & I[-U_{10}e^{(-\beta_0-\beta_1x_1-\beta_2w_0-\beta_3x_1w_0)}-U_{20}e^{(-\beta_0-\beta_1x_2-\beta_2w_0-\beta_3x_2w_0)}- \\ & U_{11}e^{(-\beta_0-\beta_1x_1-\beta_2w_1-\beta_3x_1w_1)}-U_{21}e^{(-\beta_0-\beta_1x_2-\beta_2w_1-\beta_3x_2w_1)}]+ \\ & (I-1)[-U_{10}e^{(-\beta_0-\beta_1x_1-\beta_2w_0)}-U_{20}e^{(-\beta_0-\beta_1x_2-\beta_2w_0)}- \\ & U_{11}e^{(-\beta_0-\beta_1x_1-\beta_2w_1)}-U_{21}e^{(-\beta_0-\beta_1x_2-\beta_2w_1)}]\end{aligned}$$

where $n=n_{1j}+n_{2j}$ and

$$\log(\theta_{ij})=\beta_o+\beta_1x_i+\beta_2w_j+\beta_3x_iw_j$$

where

$$U_{10}=\sum_{k=1}^{n_{10}}t_{k0}+n_{20}\tau \quad U_{20}=\sum_{k=1}^{n_{20}}(t_{k0}-\tau) \quad U_{11}=\sum_{k=1}^{n_{11}}t_{k1}+n_{21}\tau \quad U_{21}=\sum_{k=1}^{n_{21}}(t_{k1}-\tau).$$

To find the maximum likelihood estimates, $\hat{\beta}_0,\hat{\beta}_1,\hat{\beta}_2$, and $\hat{\beta}_3$, for the model parameters β_0,β_1,β_2, and β_3 we need to solve the following system of equations:

$$\begin{aligned}\frac{\partial\log L(\beta_0,\beta_1,\beta_2,\beta_3)}{\partial\beta_0} = {} & -n+I[U_{10}e^{(-\beta_0-\beta_1x_{o1}-\beta_2w_0-\beta_3x_1w_0)}+U_{20}e^{(-\beta_0-\beta_1x_{02}-\beta_2w_0-\beta_3x_2w_0)}+ \\ & U_{11}e^{(-\beta_0-\beta_1x_{o1}-\beta_2w_1-\beta_3x_1w_1)}+U_{21}e^{(-\beta_0-\beta_1x_{12}-\beta_2w_1-\beta_3x_2w_1)}]+ \\ & (I-1)[U_{10}e^{(-\beta_0-\beta_1x_{o1}-\beta_2w_0-\beta_3x_1w_0)}+U_{20}e^{(-\beta_0-\beta_1x_{02}-\beta_2w_0-\beta_3x_2w_0)}+ \\ & U_{11}e^{(-\beta_0-\beta_1x_{o1}-\beta_2w_1-\beta_3x_1w_1)}+U_{21}e^{(-\beta_0-\beta_1x_{12}-\beta_2w_1-\beta_3x_2w_1)}]=0\end{aligned}$$

$$\begin{aligned}\frac{\partial\log L(\beta_0,\beta_1,\beta_2,\beta_3)}{\partial\beta_1} = {} & -(n_{10}x_1+n_{20}x_2+n_{11}x_1+n_{21}x_2)+I[x_1U_{10}e^{(-\beta_0-\beta_1x_{01}-\beta_2w_0-\beta_3x_1w_0)}+ \\ & x_2U_{20}e^{(-\beta_0-\beta_1x_{02}-\beta_2w_0-\beta_3x_2w_0)}+x_1U_{11}e^{(-\beta_0-\beta_1x_{11}-\beta_2w_1-\beta_3x_1w_1)}+ \\ & x_2U_{21}e^{(-\beta_0-\beta_1x_{12}-\beta_2w_1-\beta_3x_2w_1)}]+(I-1)[x_1U_{10}e^{(-\beta_0-\beta_1x_{01}-\beta_2w_0-\beta_3x_1w_0)}+ \\ & x_2U_{20}e^{(-\beta_0-\beta_1x_{02}-\beta_2w_0-\beta_3x_2w_0)}+x_1U_{11}e^{(-\beta_0-\beta_1x_{11}-\beta_2w_1-\beta_3x_1w_1)}+ \\ & x_2U_{21}e^{(-\beta_0-\beta_1x_{12}-\beta_2w_1-\beta_3x_2w_1)}]=0\end{aligned}$$

Kansas State University

$$
\begin{aligned}
\frac{\partial \log L(\beta_0,\beta_1,\beta_2,\beta_3)}{\partial \beta_2} = & -(n_{10}w_0 + n_{10}w_0 + n_{11}w_1 + n_{21}w_1) + I[w_0U_{10}e^{(-\beta_0-\beta_1x_{01}-\beta_2w_0-\beta_3x_1w_0)} + \\
& w_0U_{20}e^{(-\beta_0-\beta_1x_{02}-\beta_2w_0-\beta_3x_1w_0)} + w_1U_{11}e^{(-\beta_0-\beta_1x_{11}-\beta_2w_1-\beta_3x_1w_1)} + \\
& w_1U_{21}e^{(-\beta_0-\beta_1x_{12}-\beta_2w_1-\beta_3x_2w_1)}] + (I-1)[w_0U_{10}e^{(-\beta_0-\beta_1x_{01}-\beta_2w_0-\beta_3x_1w_0)} + \\
& w_0U_{20}e^{(-\beta_0-\beta_1x_{02}-\beta_2w_0-\beta_3x_1w_0)} + w_1U_{11}e^{(-\beta_0-\beta_1x_{11}-\beta_2w_1-\beta_3x_1w_1)} + \\
& w_1U_{21}e^{(-\beta_0-\beta_1x_{12}-\beta_2w_1-\beta_3x_2w_1)}] = 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \log L(\beta_0,\beta_1,\beta_2,\beta_3)}{\partial \beta_3} = & -(n_{10}x_1w_0 + n_{20}x_2w_0 + n_{11}x_1w_1 + n_{21}x_2w_1) + \\
& I[x_1w_0U_{10}e^{(-\beta_0-\beta_1x_{01}-\beta_2w_0-\beta_3x_1w_0)} + x_2\,w_0U_{20}e^{(-\beta_0-\beta_1x_{02}-\beta_2w_0-\beta_3x_1w_0)} + \\
& x_1w_1U_{11}e^{(-\beta_0-\beta_1x_{11}-\beta_2w_1-\beta_3x_1w_1)} + x_2\,w_1U_{21}e^{(-\beta_0-\beta_1x_{12}-\beta_2w_1-\beta_3x_2w_1)}] + \\
& (I-1)[x_1w_0U_{10}e^{(-\beta_0-\beta_1x_{01}-\beta_2w_0-\beta_3x_1w_0)} + x_2\,w_0U_{20}e^{(-\beta_0-\beta_1x_{02}-\beta_2w_0-\beta_3x_1w_0)} + \\
& x_1w_1U_{11}e^{(-\beta_0-\beta_1x_{11}-\beta_2w_1-\beta_3x_1w_1)} + x_2\,w_1U_{21}e^{(-\beta_0-\beta_1x_{12}-\beta_2w_1-\beta_3x_2w_1)}] = 0
\end{aligned}
$$

The Fisher information matrix is obtained by taking the negative expected values of the second partial and mixed partial derivatives of $\log L(\beta_0,\beta_1,\beta_2,\beta_3)$ with respect to $\beta_0,\beta_1,\beta_2, and\ \beta_3$. The Fisher Information matrix is used to determine the optimal value of τ. Running the test with optimal τ minimizes the asymptotic variance of the log mean lifetime at the design stress. The Fisher information matrix is:

$$
F = n\begin{bmatrix}
A_{10}+A_{20}+A_{11}+A_{21} & \begin{pmatrix} A_{10}x_1 + A_{20}x_2 + \\ A_{11}x_1 + A_{21}x_2 \end{pmatrix} & \begin{pmatrix} A_{10}w_0 + A_{20}w_0 + \\ A_{11}xw_1 + A_{21}w_1 \end{pmatrix} & \begin{pmatrix} A_{10}x_1w_0 + A_{20}x_2w_0 + \\ A_{11}x_1w_1 + A_{21}x_2w_1 \end{pmatrix} \\
\begin{pmatrix} A_{10}x_1 + A_{20}x_2 + \\ A_{11}x_1 + A_{21}x_2 \end{pmatrix} & \begin{pmatrix} A_{10}x_1^2 + A_{20}x_2^2 + \\ A_{11}x_1^2 + A_{21}x_2^2 \end{pmatrix} & \begin{pmatrix} A_{10}x_1w_0 + A_{20}x_2w_0 + \\ A_{11}x_1w_1 + A_{21}x_2w_1 \end{pmatrix} & \begin{pmatrix} A_{10}x_1^2w_0 + A_{20}x_2^2w_0 + \\ A_{11}x_1^2w_1 + A_{21}x_2^2w_1 \end{pmatrix} \\
\begin{pmatrix} A_{10}w_0 + A_{20}w_0 + \\ A_{11}xw_1 + A_{21}w_1 \end{pmatrix} & \begin{pmatrix} A_{10}x_1w_0 + A_{20}x_2w_0 + \\ A_{11}x_1w_1 + A_{21}x_2w_1 \end{pmatrix} & \begin{pmatrix} A_{10}w_0^2 + A_{20}w_0^2 + \\ A_{11}w_1^2 + A_{21}w_1^2 \end{pmatrix} & \begin{pmatrix} A_{10}x_1\,w_0^2 + A_{20}x_2\,w_0^2 + \\ A_{11}x_1\,w_1^2 + A_{21}x_2\,w_1^2 \end{pmatrix} \\
\begin{pmatrix} A_{10}x_1w_0 + A_{20}x_2w_0 + \\ A_{11}x_1w_1 + A_{21}x_2w_1 \end{pmatrix} & \begin{pmatrix} A_{10}x_1^2w_0 + A_{20}x_2^2w_0 + \\ A_{11}x_1^2w_1 + A_{21}x_2^2w_1 \end{pmatrix} & \begin{pmatrix} A_{10}x_1\,w_0^2 + A_{20}x_2\,w_0^2 + \\ A_{11}x_1\,w_1^2 + A_{21}x_2\,w_1^2 \end{pmatrix} & \begin{pmatrix} A_{10}x_1^2w_0^2 + A_{20}x_2^2w_0^2 + \\ A_{11}x_1^2w_1^2 + A_{21}x_2^2w_1^2 \end{pmatrix}
\end{bmatrix}
$$

Where $\left(A_{10} + A_{11}\right) = 1 - e^{\left(-\frac{\tau}{\theta_{1j}}\right)}$ A_{10}, A_{11} are the probabilities that a test unit fails while at stress x_1 for each covariate group $\left(A_{20} + A_{21}\right) = 1 - \left(A_{10} + A_{11}\right)$ A_{20}, A_{21} are the probabilities that a test unit fails at stress x_2 for each group, and $A_{10} + A_{20} + A_{11} + A_{21} = 1$

4. A Numerical Example

Suppose a researcher wishes to test the two brands of twine as described earlier in Example 2. The stress could be standardized so that the design stress for the twine, 100%, is represented by $x_0 = 0$, the low stress level, 200%, by $x_1 = 1$, and the high stress, 400%, by $x_1 = 2$. The stress values will be model parameters that would have to be determined by an engineer based on how the baler twine reacts to increased stress. The τ is a time when we would expect 50 percent of the samples to have failed at low stress. The quantitative variables, $w_0 = 0$ and $w_1 = 1$ are used to distinguish between the brands of twine within the formula. If we were running an actual test we might take 20 samples of each brand, 20 pieces of CO-OP twine and 20 pieces of Heston twine, and put all 40 samples under a stress of 200% of their maximum design stress. We would note the time that each sample failed until we reach the predetermined time to change the stress to 400% of design stress noting the failure times until all samples failed. Because this type of research has not yet been done in agriculture, for this paper, we created a computer generated simulation, a brief description of that simulation follows.

A simulation of a simple step-stress test with covariate and interaction was run with a sample size of forty units, twenty from each covariate group. The following values were assumed for the simulation. The same value for τ is used for each covariate group.

$$x_0 = 0, x_1 = 1, x_2 = 2$$
$$\beta_0 = 0, \beta_1 = 1, \beta_2 = 0.01, \beta_3 = 0.02$$
$$w_0 = 0, w_1 = 1$$
$$\tau = 2.50$$

To run the simulation two groups of twenty random numbers between zero and one were generated to represent the probabilities that a test unit failed at time *t*. The equation:

$$t = \begin{cases} -\theta_{1j} \ln[1 - G(t)] & 0 < t < \tau \\ \tau - \dfrac{\theta_{2j}}{\theta_{1j}}\tau - \theta_{2j} \ln[1 - G(t)] & \tau \le t \le \infty \end{cases}$$
$$\log(\theta_{ij}) = x_i + 0.01w_j + 0.02x_i w_j$$

was used to generate the simulated failure times listed in table one. The values from this simulation allow the solving for $\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$, the maximum likelihood estimates based on the data. The maximum likelihood estimates of the model parameters are:

$$\hat{\beta}_0 = 0.474, \hat{\beta}_1 = 0.711, \hat{\beta}_2 = -0.080, \hat{\beta}_3 = -0.156.$$

The optimum time to change the stress from the lower accelerated level to the higher one is the time that minimizes the asymptotic variance of the log mean failure time at the design stress. The asymptotic variance multiplied by the sample size n is given by the equation:

$$nAV = n(1,0,1,0)F^{-1}(1,0,1,0)^T$$

Mathematica was used to find the value of τ that minimizes nAV to determine the optimal time for G; that optimal τ was found to be 3.08. Figure 1 gives evidence that the value $\tau = 3.08$ is a unique solution for the minimum value of nAV.

5. Statistical Analysis of the Results

Tests of hypotheses about parameters of the model can be obtained by using the likelihood ratio method. An important inference problem concerning the regression coefficients $(\beta_0, \beta_1, \beta_2, \beta_3)$ is the test of hypothesis $H_0 : \beta_3 = 0$ against $H_1 : \beta_3 \neq 0$. To test H_0 against H_1 one can use the likelihood ratio statistic Khamis, [3]

$$\Lambda = -2 \log \left[\frac{L(\tilde{\beta}_0, \tilde{\beta}_1, \tilde{\beta}_2, 0)}{L(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)} \right]$$

where $\tilde{\beta}_0, \tilde{\beta}_1, \tilde{\beta}_2$ are the MLEs of $\beta_0, \beta_1, \beta_2$ under $H_0, \beta_3 = 0$, and $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3$, are the unrestricted MLEs. Large values of Λ provide evidence against H_0, and approximate significance levels can be calculated by using the fact that in large samples Λ is approximately distributed as χ^2 under H_0. Using the data from the previous simulations a value for Λ can be obtained. The data from Table 1 and the log likelihood equation were used along with the MLEs.

$$\Lambda = -2[\log L(\tilde{\beta}_0, \tilde{\beta}_1, \tilde{\beta}_2, 0) - \log L(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)]$$
$$\Lambda = -2[-102.299 - (-95.345)]$$
$$\Lambda = 13.908$$

With one degree of freedom $\chi^2_{(0.05)} \approx 3.841$, $\alpha = 0.05$ significance level, the value $\Lambda = 13.908$ is significantly larger than the $\chi^2_{(0.05)}$ with one degree of freedom value so we reject the hypothesis $H_o : \beta_3 = 0$. This indicates that the interaction of the covariate with the stress level is significant.

6. Conclusion.

Accelerated life testing has long been accepted as a reliable method for testing product life but the interaction of a covariate with stress level is a previously untested idea. In this study the differences between the covariate subgroups was kept small as was the interaction between the covariate subgroups and the stress levels. This was done to reflect the use of this model in industrial or agricultural testing where the covariate subgroups could be circuit boards made to the same specifications by different suppliers or grasses of the same variety but one containing an engineered genetic mutation, which would allow one variety to be unaffected by a specific herbicide. The exponential model with covariates with interaction between the covariate subgroups and the stress level can be used as a predictor of the life of the elements of the subgroups. Further work in this area might include extension to more than two covariate groups, and using distribution models other than the exponential.

Notation

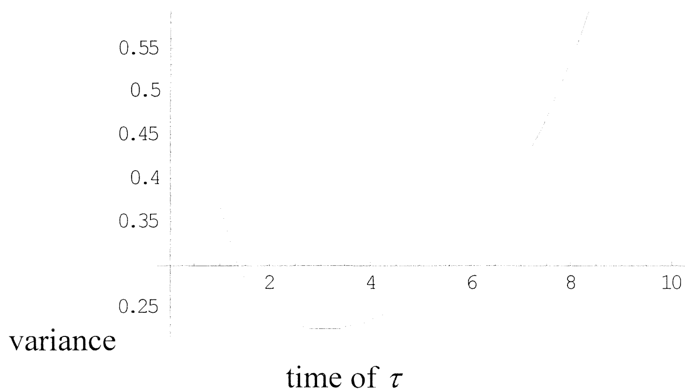
x_0, x_1, x_2	design, low, and high stress levels
w_j	covariate group; $j = 0$ and $w_j = 0$ for covariate group 1, $j = 1$ and $w_j = 1$ for covariate group 2
$x_i w_j$	the interaction between stress level and covariate group
θ_{ij}	scale parameter for stress x_i in covariate group w_j
n	total number of units in the test group
n_{ij}	number of units that failed at stress level *i* in covariate group *j*, n_i if no covariate
t_{ij}	time of unit failure at stress level *i* and covariate group *j*, t_i if no covariate
τ	time of stress change from x_1 to x_2
τ^*	optimal time to change stress from x_1 to x_2
β	model parameters
$\hat{\beta}$	maximum likelihood estimates of model parameters
AV	asymptotic variance of the log mean time at design stress
cdf	cumulative distribution function
G(t)	cdf of a test unit under simple step-stress test
MLE	maximum likelihood estimate
I	group indicator variable *I=1* for group 1 and *I=0* for group 2

Table 1 Simulated failure time data for each covariate group

Group 1	Failure Times	Group 2	Failure Times
Low stress $x_1 = 1$	0.085 0.271 0.283 0.571 1.095 1.346 1.437 1.670 1.800 2.734 2.443	Low stress $x_1 = 1$	0.058 0.058 0.330 0.511 0.517 0.946 1.212 1.430 1.516 1.845 1.953 2.472

High stress $x_2 = 2$	2.703 3.239 3.418 4.311 5.532 12.452 14.792 14.943 21.202	High stress $x_2 = 2$	2.562 2.906 4.217 4.622 4.817 8.340 10.358 17.963

Figure 1 Asymptotic variance as a function of τ



7. References

[1] W. Nelson, 'Accelerated Testing Statistical Models, Test Plans, and Data Analysis', Wiley, New York, 1990.

[2] D.S. Bia, M.S. Kim, S.H. Lee, 'Optimum simple step-stress accelerated life test with censoring,' IEEE Transactions on Reliability, Vol. R-38, December, 1989.

[3] I.H. Khamis, 'Multiple Step-Stress Testing,' unpublished PhD Thesis, Kansas State University, 1996.