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We recommend the use of Energy (E) and, or, Henze-Zirkler (HZ) tests for testing assumption of multivariate normality of data before inferential analysis could be carried out.

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Effect of Correlations on Type 1 Error Rates of Some Multivariate Normality Tests

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ABSTRACT

Normality assumption of multivariate data is a prerequisite to the use of multivariate statistical data analysis methods before inference could be valid and reliable. Tests developed to validate this assumption including Doornik-Harsen (DH), Shapiro-Francia (SF), Mardia Skewness (MS), Mardia Skewness for small sample (MSS) and Kurtosis (MK), Skewness (S) and Kurtosis(K), Shapiro-Wilk(SW), Royston (R), Desgagne-Micheaux (DM), Henze-Zirkler (HZ), Energy (E), Gel-Gastwirth (GG) and Bontemps-Meddahi (BM) tests often result into different conclusions. These differences can be misleading. Consequently, this paper examined the effect of correlations on the Type 1 error rates of multivariate tests of normality. Monte Carlo experiments were conducted one thousand (1000) times taking into consideration the dimensions, correlations and sample sizes of the multivariate data. A test is affected by correlation if its estimated Type 1 error rate changes as correlation changes. A test is considered good if its estimated error rate approximates the true error rate and best if the number of times it approximates the estimated error rate when counted over the levels of correlations, sample sizes and levels of significance is the highest, the mode. Results show that Type 1 error rates of DH, SF, SW, R, DM, GG and BM tests are affected by correlations and are relatively not good; whereas the Type 1 error rates of HZ, MS, MK, MSS, S, K and E tests are not only unaffected by correlations but are also relatively good. Consequently, MS, R, MSS, HZ and E tests have good Type 1 error rates but that of E and HZ tests are best. They are therefore recommended for practitioners.

Key Words: Correlation, Level of Significance, Type 1 error rate, Multivariate Normality test

1.0 Introduction

Multivariate inferential statistical methods including Hotelling's T^2 tests, Multivariate Analysis of Variance (MANOVA), Multivariate Analysis of Covariance (MANCOVA), Canonical Correlation, Multivariate Regression Analysis and Discriminant Analysis depend on the assumption of multivariate normality of data before inference about their parameters could be valid and reliable.

For instance, the residuals from a multivariate linear regression model are required to be normally

distributed for inference on the parameters to be valid. If they are not normally distributed then inference about the parameters are inappropriate [1].

Several tests of multivariate normality including: Doornik-Harsen (DH) [2], Shapiro-Francia (SF) [3], Mardia Skewness (MS), Mardia Skewness for small sample (MSS) and Kurtosis (MK) [4], Skewness (S) and Kurtosis (K) [5], Shapiro-Wilk (SW) [6], Royston (R) [7], Desgagne-Micheaux (DM) [8], Henze-Zirkler (HZ) [9], Energy (E) [10], Gel-Gastwirth (GG) [11] and Bontemps-Meddahi (BM) [12] have therefore been developed.

A major problem encountered with the use of these tests is that their results are seldom the same. A probable cause may be that the strength of inter-correlations that exist between the multivariate data affects the performance of some of these multivariate tests of normality. Consequently, the study was carried out to examine the effect of correlations on the Type 1 error rates of multivariate tests of normality so as to identify the stable ones and recommend them to users appropriately.

2.0 Review of Multivariate Tests of Normality

Thorough study of the literature reveals that several developed tests for assessing multivariate normality (MVN) exist. These tests were reviewed in [13] as follows.

2.1 Desgagne-Micheaux Multivariate Normality Test

Desgagné, Micheaux and Leblanc [8] proposed a new multivariate normality tests based on the distribution defined as:

$$g(x; \theta) = k(\theta) \ell^{-\frac{1}{2}|x|^{\theta_1}} (1 + |x|)^{-\theta_2} (\log(\ell + |x|))^{-\theta_3} \quad (1)$$

where $\theta = (\theta_1, \theta_2, \theta_3)'$, the vector of parameters, and $k(\theta)$ is the normalizing constant. $\theta_1 \in \mathbb{R}^+$, $x, \theta_2, \theta_3 \in \mathbb{R}$.

The proposed multivariate normality test according to [8] is given as:

$$R_n = n r_n(\bar{X}_n, S_n)^T \left(J_0 - \frac{1}{2} \nu_0 \nu_0^T \right)^{-1} r_n(\bar{X}_n, S_n) \sim \chi_3^2 \quad (2)$$

Where $r_n(\bar{X}_n, S_n) = r_n(\mu, \sigma) - \frac{1}{2}(1 - T_n)\nu_0 + O_p(n^{-1/2})1_3$, $r_n(\mu, \sigma) = \frac{1}{n} \sum_{i=1}^n d_0(Y_i)$, $Y_i = \frac{X_i - \mu}{\sigma}$

$$T_n = \frac{1}{n} \sum_{i=1}^n Y_i^2, 1_3 = (1 \ 1 \ 1)^T, J_0 = E_0[d_0(Y)d_0(Y)^T], d_0(Y) = \frac{\partial}{\partial \theta} \log g(y; \theta) \Big|_{\theta = (2 \ 0 \ 0)^T}$$

$$\nu_0 = -E_0[Y^2 d_0(Y)]$$

2.2 Mardia's Measures of Multivariate Kurtosis and Skewness

Let $X_1, X_2, \dots, X_p$ be independent n-dimensional random vectors of an identical distribution. Let p , $\bar{X}_n$ and S be the multivariate dimension, mean and covariance matrix. That is $\bar{X}_n = n^{-1} \sum_{i=1}^n X_i$ and

$S = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X}_n)(X_i - \bar{X}_n)^T$. Then Mardia's measure of multivariate kurtosis and skewness proposed by [14] are as follows:

$$k = \frac{1}{n} \sum_{i=1}^n \left[(X_i - \bar{X}_n)^T S^{-1} (X_i - \bar{X}_n) \right]^2$$

$$\frac{k - [p(p+2)(n-1)/(n+1)]}{[8p(p+2)/n]^{0.5}} \sim N(0,1) \quad (3)$$

$$s = \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n [(X_i - \bar{X}_n)^T S^{-1} (X_j - \bar{X}_n)]^3$$

$$\frac{nS}{6} \sim \chi_{p(p+1)(p+2)/6}^2 \quad (4)$$

2.3 Henze-Zirkler Test

The test proposed by Henze and Zirkler in [15] is discussed as follows:

$$T_{n,\beta} = \frac{1}{n} \sum_{k=1}^n \sum_{j=1}^n \left[\ell \left(\frac{-\beta^2 \|Y_j - Y_k\|^2}{2} \right) \right] - 2(1 + \beta^2)^{-p/2} \times \sum_{j=1}^n \left[\ell \left(\frac{-\beta^2 \|Y_j\|^2}{2(1 + \beta^2)} \right) \right] + n(1 + 2\beta^2)^{-p/2} \quad (5)$$

where $\|Y_j - Y_k\|^2 = (X_j - X_k)^T S^{-1} (X_j - X_k)$, $\|Y_j\|^2 = (X_j - \bar{X}_n)^T S^{-1} (X_j - \bar{X}_n)$

$$\beta = \frac{1}{\sqrt{2}} \left(\frac{2p+1}{4} \right)^{1/(p+4)} n^{1/(p+4)}, T_{n,\beta} \sim \ln N(E[T_\beta], \text{Var}[T_\beta])$$

$$E[T_\beta] = 1 - (1 + 2\beta^2)^{-p/2} \left[\frac{1 + p\beta^2}{1 + 2\beta^2} + \frac{p(p+2)\beta^4}{2(1 + 2\beta^2)^2} \right]$$

$$\text{Var}[T_\beta] = 2(1 + 4\beta^2)^{-p/2} + 2(1 + 2\beta^2)^{-p} \left[\frac{1 + 2p\beta^4}{(1 + 2\beta^2)^2} + \frac{3p(p+2)\beta^8}{4(1 + 2\beta^2)^4} \right]$$

$$- 4w(\beta)^{-p/2} \left[1 + \frac{3p\beta^4}{2w(\beta)} + \frac{p(p+2)\beta^8}{2w(\beta^2)} \right]$$

where $w(\beta) = (1 + \beta^2)(1 + 3\beta^2)$.

2.4 Royston's H Test

Royston's H test [7] is defined as

$$H = \xi \sum_j \frac{R_j}{p} \sim \chi_\xi^2 \quad (6)$$

where $\xi = \frac{p}{[1 + (p-1)\bar{c}]}$, $R_j = \left\{ \Phi^{-1} \left[\frac{1}{2} \Phi \left\{ \left((1 - W_j)^2 - \mu \right) / \sigma \right\} \right] \right\}^2$, W_j is the Shapiro-Wilk statistic.

λ, μ , and σ are determined from the polynomial approximations given in [16].

2.5 The Energy Test for Multivariate Normality

The multivariate energy test proposed by Szekely and Rizzo [10] is defined as follows:

$$E = n \left[\frac{2}{n} \sum_{i=1}^n E \|Y_i^* - Z\| - E \|Z - Z^T\| - \frac{1}{n^2} \sum_{i=1}^n \|Y_i^* - Y_j^*\| \right] \quad (7)$$

where $Y_i^*, i = 1, 2, \dots, n$ is the standardized sample, Z and Z^T are independently and identically distributed (i.i.d) p -variate standard normal random vectors.

2.6 Generalized Shapiro-Wilk Test

The generalized Shapiro-Wilk test according to Czeslaw [17] is given as:

$$W_p = \frac{\left\{ \sum_{i=1}^h a_{(i,n)} (U_{(n-i+1)} - U_{(i)}) \right\}^2}{(X_m - \bar{X})^T A^{-1} (X_j - \bar{X})^T} \quad (8)$$

where $U_j = (X_m - \bar{X})^T A^{-1} (X_j - \bar{X})$ for $j = 1, 2, \dots, n$, $A = \sum_{j=1}^n (X_j - \bar{X})(X_j - \bar{X})^T$, $h = \frac{n}{2}$ if n is even and $h = \frac{(n-1)}{2}$ if n is odd, $a_{(i,n)} (i = 1, 2, \dots, h)$.

The test could be compared with a quantile of the rank α of the Shapiro-Wilk distribution as thus; Reject the hypothesis if $W_p < W_p^\alpha$ where the critical value is W_p^α [17].

2.7 Doornik-Harsen Test

Doornik and Harsen test [2] is discussed as follows: compute univariate skewness and kurtosis of each normally distributed transformed n -vector of observations

$$\frac{nB_1'B_1}{6} + \frac{n(B_2 - 3t)'(B_2 - 3t)}{24} \sim \chi_a^2(2p) \quad (9)$$

where $B_1' = (\sqrt{b_{11}}, \dots, \sqrt{b_{1p}})$, $B_2' = (b_{21}, \dots, b_{2p})$ and t is a p -vector of ones. After the transformation of the data to independent normals, the univariate test is then applied to each dimension [2]. Let $Z_1' = (z_{11}, \dots, z_{1p})$ and $Z_2' = (z_{21}, \dots, z_{2p})$. Then the proposed test is:

$$E_p = Z_1'Z_1 + Z_2'Z_2 \sim \chi_a^2(2p) \quad (10)$$

2.8 Skewness (S) and Kurtosis (K) tests

The proposed tests of multivariate kurtosis and skewness by [5] are given below. The test for kurtosis is:

$$W = \left[Tr((C_1^{-1}C_2)^2) - \frac{1}{p} Tr^2(C_1^{-1}C_2) \right] + \frac{1}{p} [Tr(C_1^{-1}C_2) - p]^2 \sim \chi_{p(p+1)/2}^2 \quad (11)$$

C_1 and C_2 are two separate scatter matrices equipped with the correction factor.

The test for skewness is:

$$U = (T_1 - T_2)^T C^{-1} (T_1 - T_2) \sim \chi_p^2 \quad (12)$$

where T_1 and T_2 are two separate location vectors, and C a scatter matrix.

2.9 The Bontemps-Meddahi test

The family of normality tests proposed by Bontemps and Meddahi [12] is give as:

$$BM_{3-d} = \sum_{k=3}^d \left(\frac{1}{\sqrt{n}} \sum_{i=1}^n H_k(z_i) \right)^2 \sim \chi_{d-2}^2 \quad (13)$$

where $H_k(\cdot)$ is the k th order normalized Hermite polynomial and $z_i = \frac{(x_i - \bar{x})}{s}$.

3.0 Methodology

The investigation of how correlations affect the Type 1 error rates of the multivariate tests of normality are examined via Monte Carlo study in connection with the equation proposed in [18] defined according to equation 14. For example with $p = 5$, the equation becomes:

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} + \begin{bmatrix} \gamma_{11} & 0 & 0 & 0 & 0 \\ \gamma_{12} & \gamma_{22} & 0 & 0 & 0 \\ \gamma_{13} & \gamma_{23} & \gamma_{33} & 0 & 0 \\ \gamma_{14} & \gamma_{24} & \gamma_{34} & \gamma_{44} & 0 \\ \gamma_{15} & \gamma_{25} & \gamma_{35} & \gamma_{45} & \gamma_{55} \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{bmatrix}$$

$$X_1 = \mu_1 + \gamma_{11}Z_1$$

$$X_2 = \mu_2 + \gamma_{12}Z_1 + \gamma_{22}Z_2$$

$$X_3 = \mu_3 + \gamma_{13}Z_1 + \gamma_{23}Z_2 + \gamma_{33}Z_3$$

$$X_4 = \mu_4 + \gamma_{14}Z_1 + \gamma_{24}Z_2 + \gamma_{34}Z_3 + \gamma_{44}Z_4$$

$$X_5 = \mu_5 + \gamma_{15}Z_1 + \gamma_{25}Z_2 + \gamma_{35}Z_3 + \gamma_{45}Z_4 + \gamma_{55}Z_5$$

where

$$\gamma_{11} = \sigma_1, \gamma_{12} = \rho_{12}\sigma_2, \gamma_{13} = \rho_{13}\sigma_3, \gamma_{14} = \rho_{14}\sigma_4, \gamma_{15} = \rho_{15}\sigma_5$$

$$\gamma_{22} = \sqrt{\sigma_2^2 - \gamma_{12}^2}, \gamma_{23} = \frac{\rho_{23}\sigma_2\sigma_3 - \gamma_{12}\gamma_{13}}{\gamma_{22}}, \gamma_{24} = \frac{\rho_{24}\sigma_2\sigma_4 - \gamma_{12}\gamma_{14}}{\gamma_{22}}$$

$$\gamma_{25} = \frac{\rho_{25}\sigma_2\sigma_5 - \gamma_{12}\gamma_{15}}{\gamma_{22}}, \gamma_{33} = \sqrt{\sigma_3^2 - \gamma_{13}^2 - \gamma_{23}^2}, \gamma_{34} = \frac{\rho_{34}\sigma_3\sigma_4 - \gamma_{13}\gamma_{14} - \gamma_{23}\gamma_{24}}{\gamma_{33}}$$

$$\gamma_{35} = \frac{\rho_{35}\sigma_3\sigma_5 - \gamma_{13}\gamma_{15} - \gamma_{23}\gamma_{25}}{\gamma_{33}}, \gamma_{44} = \sqrt{\sigma_4^2 - \gamma_{14}^2 - \gamma_{24}^2 - \gamma_{34}^2}$$

$$\gamma_{45} = \frac{\rho_{45}\sigma_4\sigma_5 - \gamma_{14}\gamma_{15} - \gamma_{24}\gamma_{25} - \gamma_{34}\gamma_{35}}{\gamma_{44}} \text{ and } \gamma_{55} = \sqrt{\sigma_5^2 - \gamma_{15}^2 - \gamma_{25}^2 - \gamma_{35}^2 - \gamma_{45}^2}$$

The parameters for the Monte Carlo simulation study are: seven levels of inter-correlations namely $\rho_{ij} = \rho = 0, 0.3, 0.6, 0.8, 0.9, 0.95, \text{ and } 0.99 \forall i=1,2,\dots, p \text{ and } j=1,2,\dots, p$, four levels of dimension, $p = 2, 3, 4 \text{ and } 5$, eight sample sizes $n = 10, 20, 30, 50, 75, 100, 150 \text{ and } 300$, three levels of significance, $\alpha = 0.1, 0.05 \text{ and } 0.01$.

The simulation procedure is according to the procedure outlined in [19]. We generated 1000 (Replications (K)) set of multivariate normally distributed samples using the specified values of p and n from R-program environment [20]. The generated data were then subjected to the multivariate normality tests and the associated p-values as a result of the tests were documented at each stage of the replications. Let

$$S_i = \begin{cases} 1, & \text{if } p\text{-value} < \alpha \\ 0, & \text{Otherwise} \end{cases} \quad \forall i = 1, 2, \dots, K \quad (15)$$

Define $S = \sum_{i=1}^K S_i$, then the estimated Type 1 error rate is given as:

$$E_\alpha = \frac{S}{K} \quad (16)$$

The method was repeated until all the parameters for the simulation were completely utilized. A multivariate test of normality is affected by correlation if its estimated Type 1 error rate changes as level of correlation changes. It is stable if as correlation changes the Type 1 error rates remain the same. Furthermore, a test is considered good if its estimated error rates approximates the true error

rate, and best if its empirical Type 1 error rates approximates the true error rates most often (Highest frequency) when counted over the three levels of significance. A test with an error rate outside of ranges presented in Table 1 is considered comparatively 'not good'.

Table 1: The True Level of Significance and Their Preferred Interval

True levels of significance	Preferred interval
0.1	0.095 – 0.14
0.05	0.045 - 0.054
0.01	0.005 - 0.014

Source: [13] & [21]

4.0 Results and Discussion

The results of effect of correlations on Type 1 error rates of multivariate normality tests for the levels of significance, correlations, dimensions and sample sizes from the empirical data available in [19] are presented and discussed. The results show that Type 1 error rates of some normality tests are affected by correlations.

The results at 0.1 level of significance is presented in Figure 1 (see Table 2 for sample data). It was observed that the Type 1 error rates of DH, GG, BM and DM increase as correlations increase. Type 1 error rates of R, HZ, MS, MK, MSS, S, K, SW, SF and E are not affected by correlations. Type 1 error rates of DH, GG, BM, DM, SW and SF are comparatively not good but that of R, HZ, MS, MK, MSS, S, K, and E are comparatively good. Results from counting the number of times Type 1 error rates approximate 0.1 show that: Type 1 error rates of MS, MK, DH, S, K, SW, SF, GG, BM and DM tests are relatively not good at almost all sample sizes while that of HZ, MSS, R and E tests are reasonably good at almost all sample sizes. Therefore, R, E, MSS and HZ tests in this order perform well.

The results at 0.05 level of significance is displayed in Figure2 (see Table 3 for sample data), the results show that: the Type 1 error rates of HZ, MS, MK, R, K, MSS, S, SW, SF and E are not affected by correlations. Type 1 error rates of DM, BM, GG, and DH increase as correlations increase. As sample sizes and dimensions increase, the effect correlations on the tests Type 1 error rates becomes more pronounced. Type 1 error rates of HZ, MS, MK, R, K, MSS, S, and E revolve around 0.05. Type 1 error rates of GG, BM, DH, DM, SW and SF are comparatively not good. Results from counting the number of times Type 1 error rates approximate 0.05 revealed that: Type 1 error rates of all the tests are generally not good. Type 1 error rate of MS is good for sample size greater than or equal to 75 except at 300. E test has good Type 1 error rates for sample size 10.

The result of effect of correlations on Type 1 error rates examined at 0.01 level of significance is presented in Figure 3 (See Table 4 for sample data). It was observed that the Type 1 error rates of R, K, MSS, HZ, MS, MK, S, SW, SF and E are not affected by correlations. At lower dimensions,

Type 1 error rates of BM, GG, DM, and DH reduce and converge to 0.01 as correlations increase while the same increases at higher dimension as correlation increase. As sample sizes and dimensions increase, the effect of the correlations on the tests Type 1 error rates is more noticeable. Type 1 error rates of R, K, MSS, HZ, MS, MK, S, and E revolve around 0.01 at all sample sizes. The Type 1 error rates of BM, DH, DM, GG, SF and SW are comparatively not good at this level. Results from counting the number of times Type 1 error rates approximate 0.01 revealed that the Type 1 error rates of GG, BM, DG, SW, SF, K, DH tests are relatively not good at almost all sample sizes while that of MS is good for sample sizes 20 to 100. Also, the Type 1 error rates of MK is good for sample size of at least 50 while it is not good for sample sizes less than or equal to 300. Furthermore, HZ, S and E tests have good Type 1 error rates at almost all sample sizes but Type 1 error rates of S is not good for sample sizes less than or equal to 30 while Type 1 error rates of HZ is not good at sample size 10.

Table 5 shows the overall total number of times Type 1 error rates of the multivariate normality tests approximated true levels of significance when counted over levels of significance, dimensions and levels of correlations. A maximum count of 84 is expected from each of the tests at each sample level. The result shows that the Type 1 error rates of MS, MK, DH, S, K, SW, SF, GG, BM and DM tests are generally not good for sample sizes less than or equal to 30. Type 1 error rates of E and HZ tests are good at all sample sizes except at sample size of 10 where that of HZ deviates. Type 1 error rates of MSS test are fair at almost all sample sizes. Having further counted over sample sizes, critical analysis of the results show that MS, R, MSS, HZ and E tests have good Type 1 error rates behavior in that order. It should be noted that total overall expected maximum count is 672. The results are presented in Figure 4.

Figure 1: Comparison of Type 1 Error Rates of Multivariate Normality Tests in the Presence of Correlation at 0.1 Level of Significance

Table 2: Sample of Empirical Type 1 Error Rate for Comparison of Multivariate Normality Tests at 10% Level of Significance

Sample Size		10				20				
Test	p\rho	0	0.6	0.9	0.99	0	0.6	0.9	0.95	0.99
HZ	2	0.096	0.096	0.096	0.096	0.084	0.084	0.084	0.084	0.084
	3	0.08	0.08	0.08	0.08	0.094	0.094	0.094	0.094	0.094
	4	0.094	0.094	0.094	0.094	0.082	0.082	0.082	0.082	0.082
MS	2	0.019	0.019	0.019	0.019	0.054	0.054	0.054	0.054	0.054
	3	0.002	0.002	0.002	0.002	0.04	0.04	0.04	0.04	0.04
	4	0	0	0	0	0.033	0.033	0.033	0.033	0.033
MK	2	0	0	0	0	0.014	0.014	0.014	0.014	0.014
	3	0	0	0	0	0.016	0.016	0.016	0.016	0.016
	4	0.002	0.002	0.002	0.002	0.033	0.033	0.033	0.033	0.033
MSS	2	0.116	0.116	0.116	0.116	0.097	0.097	0.097	0.097	0.097
	3	0.09	0.09	0.09	0.09	0.124	0.124	0.124	0.124	0.124
	4	0.06	0.06	0.06	0.06	0.106	0.106	0.106	0.106	0.106
R	2	0.123	0.133	0.115	0.119	0.116	0.126	0.12	0.126	0.136
	3	0.113	0.117	0.099	0.1	0.135	0.127	0.124	0.123	0.129
	4	0.11	0.119	0.109	0.118	0.145	0.13	0.118	0.117	0.115
DH	2	0.245	0.242	0.259	0.261	0.36	0.336	0.347	0.353	0.353
	3	0.173	0.155	0.127	0.126	0.214	0.187	0.208	0.218	0.211
	4	0.291	0.206	0.126	0.098	0.412	0.325	0.2	0.185	0.172
S	2	0.004	0.004	0.004	0.004	0.044	0.044	0.044	0.044	0.044
	3	0	0	0	0	0.026	0.026	0.026	0.026	0.026
	4	0	0	0	0	0.016	0.016	0.016	0.016	0.016
K	2	0	0	0	0	0.018	0.018	0.018	0.018	0.018
	3	0	0	0	0	0.007	0.007	0.007	0.007	0.007
	4	0	0	0	0	0.004	0.004	0.004	0.004	0.004
SW	2	0.26	0.26	0.26	0.26	0.204	0.204	0.204	0.204	0.204
	3	0.586	0.586	0.586	0.586	0.493	0.493	0.493	0.493	0.493
	4	0.895	0.895	0.895	0.895	0.733	0.733	0.733	0.733	0.733
SF	2	0.346	0.346	0.346	0.346	0.268	0.268	0.268	0.268	0.268
	3	0.707	0.707	0.707	0.707	0.615	0.615	0.615	0.615	0.615
	4	0.956	0.956	0.956	0.956	0.863	0.863	0.863	0.863	0.863
E	2	0.119	0.119	0.119	0.119	0.087	0.087	0.087	0.087	0.087
	3	0.102	0.102	0.102	0.102	0.113	0.113	0.113	0.113	0.113
	4	0.104	0.104	0.104	0.104	0.097	0.097	0.097	0.097	0.097
GG	2	0.241	0.255	0.258	0.273	0.39	0.337	0.379	0.381	0.383
	3	0.158	0.145	0.123	0.107	0.225	0.19	0.195	0.196	0.188
	4	0.283	0.186	0.099	0.084	0.433	0.313	0.153	0.131	0.121
BM	2	0.108	0.098	0.124	0.13	0.266	0.219	0.245	0.236	0.25
	3	0.076	0.077	0.075	0.065	0.157	0.131	0.141	0.141	0.149
	4	0.202	0.128	0.068	0.059	0.357	0.249	0.126	0.125	0.112
DM	2	0.192	0.179	0.183	0.18	0.302	0.277	0.282	0.285	0.286
	3	0.147	0.098	0.081	0.071	0.171	0.136	0.12	0.13	0.12
	4	0.19	0.129	0.084	0.073	0.28	0.168	0.11	0.117	0.118

Source: Simulation Results available in [19]

Figure 2: Comparison of Type 1 Error Rates of Multivariate Normality Tests in the Presence of Correlation at 0.05 Level of Significance

Table 3: Sample of Empirical Type 1 Error Rate for Comparison of Multivariate Normality Tests at 5% Level of Significance

Sample Size		10				20			
Test	ρ	0	0.6	0.9	0.99	0	0.6	0.9	0.99
HZ	2	0.035	0.035	0.035	0.035	0.039	0.039	0.039	0.039
	3	0.032	0.032	0.032	0.032	0.038	0.038	0.038	0.038
	4	0.034	0.034	0.034	0.034	0.032	0.032	0.032	0.032
MS	2	0.004	0.004	0.004	0.004	0.029	0.029	0.029	0.029
	3	0	0	0	0	0.015	0.015	0.015	0.015
	4	0	0	0	0	0.011	0.011	0.011	0.011
MK	2	0	0	0	0	0.007	0.007	0.007	0.007
	3	0	0	0	0	0.001	0.001	0.001	0.001
	4	0	0	0	0	0	0	0	0
MSS	2	0.052	0.052	0.052	0.052	0.061	0.061	0.061	0.061
	3	0.042	0.042	0.042	0.042	0.064	0.064	0.064	0.064
	4	0.025	0.025	0.025	0.025	0.063	0.063	0.063	0.063
R	2	0.062	0.072	0.066	0.07	0.052	0.061	0.072	0.072
	3	0.057	0.054	0.063	0.064	0.073	0.069	0.068	0.073
	4	0.058	0.061	0.053	0.058	0.07	0.08	0.072	0.063
DH	2	0.16	0.149	0.168	0.159	0.266	0.24	0.248	0.247
	3	0.096	0.087	0.08	0.077	0.141	0.119	0.14	0.144
	4	0.206	0.143	0.075	0.054	0.322	0.217	0.122	0.108
S	2	0.001	0.001	0.001	0.001	0.021	0.021	0.021	0.021
	3	0	0	0	0	0.011	0.011	0.011	0.011
	4	0	0	0	0	0.003	0.003	0.003	0.003
K	2	0	0	0	0	0.012	0.012	0.012	0.012
	3	0	0	0	0	0.003	0.003	0.003	0.003
	4	0	0	0	0	0.002	0.002	0.002	0.002
SW	2	0.162	0.162	0.162	0.162	0.117	0.117	0.117	0.117
	3	0.44	0.44	0.44	0.44	0.335	0.335	0.335	0.335
	4	0.791	0.791	0.791	0.791	0.586	0.586	0.586	0.586
SF	2	0.217	0.217	0.217	0.217	0.172	0.172	0.172	0.172
	3	0.544	0.544	0.544	0.544	0.452	0.452	0.452	0.452
	4	0.886	0.886	0.886	0.886	0.712	0.712	0.712	0.712
E	2	0.049	0.049	0.049	0.049	0.042	0.042	0.042	0.042
	3	0.05	0.05	0.05	0.05	0.044	0.044	0.044	0.044
	4	0.058	0.058	0.058	0.058	0.036	0.036	0.036	0.036
GG	2	0.203	0.197	0.215	0.224	0.337	0.282	0.328	0.325
	3	0.127	0.104	0.096	0.081	0.173	0.138	0.149	0.15
	4	0.243	0.147	0.075	0.056	0.363	0.247	0.104	0.094
BM	2	0.08	0.071	0.093	0.088	0.189	0.179	0.186	0.183
	3	0.057	0.043	0.051	0.047	0.113	0.095	0.096	0.114
	4	0.16	0.095	0.036	0.037	0.289	0.184	0.085	0.081
DM	2	0.128	0.11	0.112	0.108	0.216	0.181	0.178	0.198
	3	0.088	0.051	0.041	0.044	0.097	0.069	0.073	0.078
	4	0.124	0.063	0.05	0.041	0.195	0.096	0.052	0.057

Source: Simulation Results available in [19]

Figure 3: Comparison of Type 1 Error Rates of Multivariate Normality Tests in the Presence of Correlation at 0.01 Level of Significance

Table 4: Sample of Empirical Type 1 Error Rate for Comparison of Multivariate Normality Tests at 1% Level of Significance

Sample Size		10				20			
Test	ρ	0	0.6	0.9	0.99	0	0.6	0.9	0.99
HZ	2	0.002	0.002	0.002	0.002	0.008	0.008	0.008	0.008
	3	0.004	0.004	0.004	0.004	0.007	0.007	0.007	0.007
	4	0.003	0.003	0.003	0.003	0.005	0.005	0.005	0.005
MS	2	0.001	0.001	0.001	0.001	0.012	0.012	0.012	0.012
	3	0	0	0	0	0.005	0.005	0.005	0.005
	4	0	0	0	0	0.001	0.001	0.001	0.001
MK	2	0	0	0	0	0.002	0.002	0.002	0.002
	3	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0
MSS	2	0.01	0.01	0.01	0.01	0.022	0.022	0.022	0.022
	3	0.003	0.003	0.003	0.003	0.012	0.012	0.012	0.012
	4	0.001	0.001	0.001	0.001	0.015	0.015	0.015	0.015
R	2	0.013	0.013	0.014	0.015	0.016	0.019	0.02	0.015
	3	0.004	0.01	0.02	0.014	0.018	0.021	0.015	0.012
	4	0.009	0.007	0.017	0.016	0.018	0.019	0.023	0.019
DH	2	0.063	0.048	0.051	0.058	0.114	0.102	0.111	0.112
	3	0.033	0.023	0.021	0.028	0.054	0.046	0.056	0.057
	4	0.076	0.041	0.019	0.022	0.157	0.094	0.048	0.039
S	2	0	0	0	0	0.005	0.005	0.005	0.005
	3	0	0	0	0	0	0	0	0
	4	0	0	0	0	0.001	0.001	0.001	0.001
K	2	0	0	0	0	0.003	0.003	0.003	0.003
	3	0	0	0	0	0.001	0.001	0.001	0.001
	4	0	0	0	0	0	0	0	0
SW	2	0.069	0.069	0.069	0.069	0.038	0.038	0.038	0.038
	3	0.204	0.204	0.204	0.204	0.131	0.131	0.131	0.131
	4	0.511	0.511	0.511	0.511	0.274	0.274	0.274	0.274
SF	2	0.072	0.072	0.072	0.072	0.045	0.045	0.045	0.045
	3	0.219	0.219	0.219	0.219	0.17	0.17	0.17	0.17
	4	0.541	0.541	0.541	0.541	0.342	0.342	0.342	0.342
E	2	0.009	0.009	0.009	0.009	0.012	0.012	0.012	0.012
	3	0.009	0.009	0.009	0.009	0.012	0.012	0.012	0.012
	4	0.007	0.007	0.007	0.007	0.008	0.008	0.008	0.008
GG	2	0.143	0.134	0.158	0.157	0.242	0.213	0.231	0.257
	3	0.084	0.059	0.066	0.051	0.1	0.085	0.095	0.099
	4	0.172	0.093	0.039	0.034	0.262	0.149	0.064	0.064
BM	2	0.048	0.038	0.042	0.052	0.124	0.107	0.105	0.106
	3	0.035	0.023	0.021	0.027	0.064	0.045	0.054	0.069
	4	0.097	0.047	0.019	0.019	0.2	0.104	0.045	0.048
DM	2	0.054	0.042	0.029	0.032	0.088	0.072	0.08	0.074
	3	0.018	0.009	0.008	0.012	0.034	0.026	0.022	0.027
	4	0.044	0.015	0.013	0.011	0.082	0.03	0.012	0.016

Source: Simulation Result available in [19]

Table 5: Total Number of Times Type 1 Error Rate Approximates True Value Over All Levels Significance, Levels Dimension and Correlation

Tests	Sample Size								Total	Rank
	10	20	30	50	75	100	150	300		
HZ	21	49	42	49	56	55	49	63	384	2
MS	0	14	28	42	56	49	49	22	260	5
MK	0	0	0	14	21	14	21	35	105	7
MSS	21	41	43	35	28	35	28	49	280	3
R	49	32	28	34	33	41	24	34	275	4
DH	12	0	0	0	0	0	0	0	12	11
S	0	7	7	21	28	28	42	28	161	6
K	0	0	14	14	0	14	14	28	84	8
SW	0	0	0	0	0	0	0	0	0	13.5
SF	0	0	0	0	0	0	0	0	0	13.5
E	77	49	42	49	49	70	63	56	455	1
GG	7	2	0	0	0	0	0	0	9	12
BM	10	5	0	0	0	0	0	0	15	10
DM	19	12	5	2	0	0	0	0	38	9

Source: Counted from Simulation Results available in [19]

Figure 4: Bar Chart Showing Total Number of Times Type 1 Error Rate Approximates True Value Over All Significance Levels, Dimensions and Correlations

5.0 Conclusion

The effect of correlations was examined on the Type 1 error rates of the multivariate tests of normality. The study revealed that the Type 1 error rates of R, DH, GG, BM and DM tests are affected by correlation while those of HZ, MS, MK, MSS, S, K, SW, SF and E tests are not affected by correlation. Type 1 error rates of tests affected by correlations are comparatively bad except that of R whereas those of tests not affected by correlation revolve around the true levels of significance except that of SW and SF. MS, R, MSS, HZ and E tests, in this order, have good Type 1 error rates. Hence, E and HZ tests are therefore identified to have best Type 1 error rates in the presence of correlations. They are therefore recommended for practitioners.

This conclusion and recommendation are limited to the fact that no standard comparison was used. However, comparisons are only based on preferred intervals and frequency of counts.

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