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Elementary Preservice Mathematics Teachers Fraction Knowledge: An Integrative Review of Research

Cody J. Perry

Fractions are an integral building block of middle elementary mathematics, and students who master fractions are better prepared for subsequent mathematics concepts like algebra and probability (Gabriel, 2016; Siegler et al., 2011). However, many people, including adults and teachers, struggle with fractions, as they attempt to build on incomplete knowledge to master rational numbers (Faulkenberry & Pierce, 2011; Tobias, 2013). From math anxiety and future engagement with STEM classes and careers to medical professionals who made dosing mistakes, difficulties with fractions result in extensive consequences (Gabriel, 2016). Although it may seem counterintuitive, one group that experiences considerable fraction issues is future teachers who will teach rational number concepts to future cohorts of children (Bruce et al., 2013). Since mathematics scores have not improved appreciably in the last 20 years, a need exists to improve fraction instruction for preservice teachers (PSTs), so future elementary students are more successful with fractions (NCES, 2019). Unfortunately, it appears that many future teachers have not mastered fractions to the point they can teach fractions without struggling or repeating the superficial approach their teachers used.

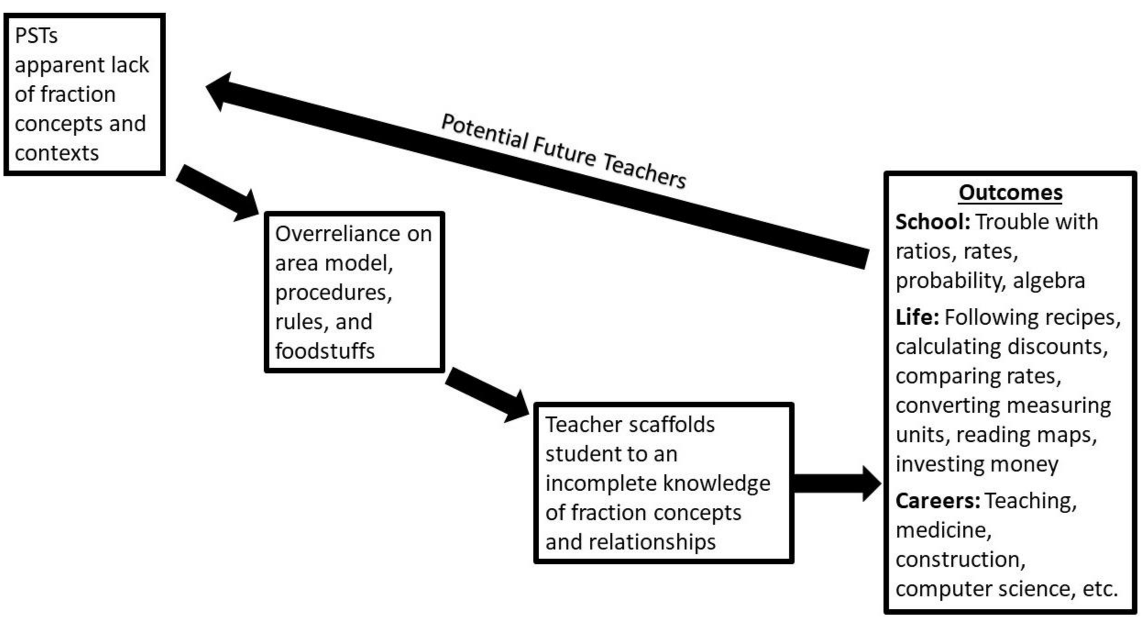
While fractions are important in innumerable contexts, teachers should be well-versed in rational number concepts, algorithms, and pedagogical approaches. However, many PSTs struggled to define a whole, find real-world examples, and explain concepts and relationships (Lo & Luo, 2012; Muzheve & Capraro, 2011; Tobias, 2013). They also struggled to understand underlying concepts and relied too much on procedures and rules (Alenazi, 2015; Sahin et al., 2020). Numerous studies also found that about half of all PSTs had difficulty with fractions and fraction concepts that should have been conquered in elementary school (Lee, 2017; Reeder & Utley, 2017; Son & Lee, 2016; Thanheiser et al., 2016). This indicates that these future educators will be unable to help their students master fractions and will enter the classroom with a limited knowledge of fractions. How can one expect elementary students to master rational numbers and improve test scores when their teachers still need to improve their own understanding and skill? Educator preparation programs (EPPs) and faculty members have a significant opportunity to address and improve PSTs’ fraction knowledge and thus their ability to teach rational numbers before they enter the classroom. Since fractions are so vital in school, STEM careers, and the real world, exploring improved fraction performance among PSTs may also benefit practicing teachers and students alike (Bruce et al., 2013; Gabriel, 2016). Thus, this integrative review of previous PST fraction research seeks to guide future studies and inform EPPs about the improvement of fraction mastery among future educators. The review was guided by the following research questions:

1. What has research revealed about PSTs' fraction knowledge and skill deficiencies?
2. What strategies may not be effective in helping one master fractions?
3. What strategies and interventions improved PSTs fraction knowledge and performance?

In completing this analysis of extant research, I hoped to develop a clearer picture of future avenues of inquiry, discover ineffective strategies, find potential strategies to help my students improve, and provide information that may help EPPs' approach to prepare elementary math teachers.

Framework

One potential reason for widespread struggles with fractions is that many teachers possess a limited knowledge of concepts and rely too much on area models, foodstuffs, rules, procedures, etc. Therefore, many teachers cannot put fractions into context or create meaningful, engaging fractions lessons (Crawford, 1996). While many educators claim to use scaffolded instruction advocated by Vygotsky, research has shown teachers often struggle to provide the necessary support required to master mathematics (Denhere et al., 2013). When a PST's knowledge is imperfect, she/he may be unable to differentiate for students' needs, and their students may also develop limited rational number knowledge. Moreover, the misapplication of educational theories, especially those of Vygotsky and Piaget, may lead to a teacher-dictated, over-structured atmosphere, where fluency is the main aim of instruction and learning (Fuson, 2009). While fluency is important, it is only one facet of fractions mastery, as conceptual understanding, productive disposition, adaptive reasoning, and strategic competence are also needed (Van de Walle et al., 2019). Many classrooms seem to adopt a strategy where the teacher shows a handful of examples before the class solves some together. Once the teacher feels the scaffolding has worked, students are asked to perform problems on their own. However, this approach does not align with Vygotsky's ideas of the Zone of Proximal Development (ZPD) being the place where a child can succeed if given proper support and (mediation) scaffolding. He thought this support should include tools, attention to language, and collaboration with peers (Vygotsky, 1978). Therefore, since some PSTs operate with limited knowledge and superficial scaffolding, they cannot provide the tools and competencies their students need to master fractions. This leads to students who continue to focus on rules, procedures, and area models, but lack a grasp of concepts, connections with decimals, percentages, etc., and real-world applications (Crawford, 1996). These students, some of whom will eventually become teachers, carry misconceptions and errors through high school, college, and beyond (Fazio et al., 2016). The figure below represents the cyclical nature of fractions instruction and learning:



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While the best course of action is to address fraction deficiencies while students are in elementary school, we first need teachers who are experts with rational numbers. To accomplish this though, we must determine the best ways to prepare current PSTs for their future classroom, and a review of research is a logical first step in identifying obstacles to mastery and promising strategies for improving fraction skills and knowledge.

Method

According to professional experience, anecdotal evidence, and extant literature, widespread weaknesses in fraction skills and knowledge exist for a range of grade levels through adulthood. A potential explanation for this phenomenon is that some teachers lack the content and pedagogical knowledge to equip their students for success. The literature revealed that the problem has not been adequately addressed yet, and PSTs who enter the classroom with limited fraction mastery hamper schools' efforts to improve fractions and their applications for many elementary students. Since the problem has not improved appreciably, we need to delve more into preventing it from happening in the future by determining recent approaches that may have been effective, so we have a good starting point to inform the next decade of research and intervention. Therefore, this integrative review began with a search of the university's database for peer-reviewed studies of the last decade from EBSCO Host, Eric, ProQuest, and the collection of journals to which the university subscribes. My initial search included 10 keywords/phrases like fraction-teaching strategies, fractions mastery, and teaching fractions. To be included, an article had to be based on empirical research, published in a peer-reviewed journal, focused on strategies for improving fraction performance, and published within the last 10 years in English. Using the above approach and criteria, I initially found 70 studies to review.

Literature Analysis and Synthesis

After compiling the 70 studies, I read and noted each article looking at the populations studied, the interventions and methodology used, the data collected, and the results and conclusions for each. I kept track of the studies by creating a spreadsheet with each of the variables above as headings along with each study's evidence of improvement for PSTs' fraction mastery. After the first reading of all 70 studies, I narrowed the review to studies of PSTs since many will be teaching fractions soon. This left me with 19 studies from the last decade that adhered to all selection criteria. After choosing to narrow the review to PSTs only, I read each study again and recorded information in a new spreadsheet with the following headings: year of publication, number of participants, research design, fraction strategies used, and results. After a third reading of each study, I divided the literature into two categories: exploratory and quasi-experimental studies. I then coded and analyzed the nine exploratory studies for discoveries they made concerning PSTs' fraction knowledge and performance. Following the analysis of those studies, I coded the quasi-experimental studies based on their major findings, the type of intervention used, suggestions for future study, etc. In addition, I grouped those 10 studies by the type of intervention used (e.g., using number lines, multiple representations) and how performance data was affected. Finally, I examined the studies and coding according to the research questions beginning with PSTs' fraction knowledge and skill deficiencies. Then, I noted strategies that seemed to be ineffective, which could be discontinued or revisited after additional research is conducted. I also reviewed PSTs' improvement data to determine which strategies were found to be the most successful. I compared performance data based on the strategy used and read the articles again to identify commonalities, differences, etc., which allowed me to identify strategies that EPPs may want to use to improve PSTs' fraction mastery before they enter the classroom.

Findings

Of the 19 studies reviewed, 9 were exploratory only, and 10 were investigations of interventions/strategies aimed at improving PSTs' fraction knowledge and/or skills. Five of the exploratory studies viewed which fraction strategies PSTs chose when solving fractions problems/tasks, two examined how PSTs had learned about fractions, and the final two studies investigated pictorial models. Of the 10 quasi-experimental studies, four focused on PSTs' conceptual knowledge rather than procedures alone, three gauged the effects of measurement and/or number lines, one explored the use of a magnitude intervention, one viewed pictorial models, and the last study employed a five-week unit that mirrored a sequence often used in elementary schools.

Obstacles to Fraction Mastery. Once the initial coding and analysis were completed, I considered the literature based on my research questions, starting with which concepts, algorithms, etc. PSTs struggled with the most. By understanding impediments to mastery, I hoped to glean avenues for future research, identify rational number concepts to address with my students, and inform EPPs' approach to preparing future teachers. The literature revealed three main issues that may have prevented PSTs from mastering fractions, with a lack of conceptual understanding as the most common (n = 5). Two studies found that PSTs focused mainly on procedures, and their conceptual knowledge was weak, their reasoning superficial, and many had developed misconceptions that would preclude them from helping their students (Reeder & Utley, 2017; Sahin et al., 2020). Although these PSTs may be able to solve problems on their own, they may struggle to teach others because they lack conceptual knowledge themselves. Whitehead and Walkowiak (2017) echoed these findings as their data showed PSTs think that explaining mathematics is rooted in restating rules detached from conceptual understanding. Finally, Alenazi's (2015) participants applied algorithms but did not understand the reason for doing it that way or how the steps truly worked. Therefore, many PSTs may be able to solve certain problems with the strategies they have memorized, but lack the necessary conceptual and pedagogical capital to offer thorough, meaningful explanations to their students.

A second deficiency in PSTs' fraction mastery was the use of questionable or incorrect strategies when working with various applications. Fazio et al. (2016) found that even after providing alternative solutions or strategies, many participants chose to use the same strategy even if it was unsuccessful the first time around. Sahin et al. (2020) discovered that PSTs made numerous mistakes when dividing fractions that included multiplying without inverting the second fraction, cross-multiplying after inverting, and incorrectly interpreting answers. In addition, PSTs who struggled knew fewer strategies to solve fraction problems than those who performed better (Whiteacre & Nickerson, 2016). Numerous strategies exist to solve a range of mathematics problems, and flexibility in the use of different approaches makes finding a solution much easier. When PSTs make procedural errors, lack understanding of multiple strategies, or apply them in the wrong context, their students will struggle to learn fractions well enough to build upon their knowledge in the future. The best strategies for solving problems will fail when applied incorrectly, so teachers must have a solid grasp of multiple methods to ensure their students have success with rational numbers.

Weaknesses related to fraction magnitude also posed problems (Whiteacre & Nickerson, 2016). Reeder and Utley (2017) found that PSTs had magnitude misconceptions as only 46% of participants were able to order three fractions incorrectly. In addition, those who did not perform well almost always relied on questionable or incorrect strategies and their failure to compare fractions based on fractions with the same numerator showed a lack of conceptual understanding (Fazio et al., 2016). In contrast, those who were most successful had a better understanding of magnitude and the distance between values (Faulkenberry & Pierce, 2011). An understanding of magnitude and one's ability to order fractions is integral to equivalent fractions and expressing them in simplest terms. Therefore, struggling with magnitude and ordering of fractions can lead to numerous challenges later and highlights the need to determine why so many PSTs do not grasp some basic fraction concepts. The most challenging fraction concepts appeared to be multiplication and division, which are more challenging if PSTs do not understand magnitude, which procedures work best, and underlying concepts like inverse operations. Thus, PSTs' hyperfocus on procedures, questionable choice of solution strategies, and misunderstandings of fraction magnitude may explain why many are unable to help their students reach an acceptable level of rational number mastery.

Most Common Fraction Challenges. Identifying common fraction challenges for PSTs was instructive, but it was also essential to look for pedagogical methods that may be ineffective, especially if they are widely used. Since many teachers appear to be limited even though they have taken numerous mathematics courses, EPPs have an opportunity to intervene before aspiring educators enter the classroom. Therefore, understanding ineffective strategies may help improve instruction and raise fraction performance for teachers and students alike. Three potentially ineffective teaching methods emerged from the literature, which included relying too much on memorizing procedures and tricks, the use of only one representation, and incorrect manipulative use. While memorization, use of the area model, and manipulatives are all facets of fraction mastery, when teachers do not understand concepts and effective pedagogy, these strategies may hinder students' understanding of fractions and their ability to solve a range of problems.

Although it is common for people to view mathematics as largely the memorization of facts and procedures, true mastery requires more. Sahin et al.'s (2020) participants learned to invert and multiply when dividing fractions, but at least 40% made mistakes when solving these problems. In addition, relying solely on procedures prevented PSTs from recognizing different contexts, led to making more errors, and many failed to recognize the errors students made (Son & Lee, 2016). Consequently, Whitehead and Walkowiak (2017) found teaching algorithms and procedures without exploring and explaining the reason each worked, contributed to PSTs' limited knowledge, which will inhibit their ability to help students reach their potential. As the article highlighted, PSTs learning to go beyond rote memorization, procedures, and rules, may have a trickle-down effect on their future students.

While relying on memorization too much may be problematic, the proper use of fraction representations is integral for PSTs and their future students. Multiple representations help build a conceptual foundation and Harvey (2012) found that many issues may have been caused by a lack of depictions, which could have been improved by using number lines. Moreover, since students think in a variety of contexts, teachers must be able to use various representations to differentiate and respond to students (Son & Lee, 2016). Furthermore, Reeder and Utley (2017) found nearly all PSTs' (90.2%) limits seemed to spring from only using the part-whole representation of fractions. Since they were aware of only one depiction, their reasoning was shallow, and their ability to work in other contexts was limited. With the diverse makeup of classrooms, teachers must be able to communicate with students in a variety of representations and contexts to meet the needs of all. Therefore, helping PSTs grasp additional models of fractions may improve differentiated instruction and help students master fractions.

Finally, while many teachers use manipulatives in the classroom, they may hinder fraction mastery when used improperly. Manipulatives like fraction and pattern blocks, cutting objects, etc. can be effective, but only if the teacher understands the appropriate applications and the manipulative is seen as a tool rather than the focus of learning (Harvey, 2012). Furthermore, pattern blocks were not effective in helping older children make mental models of fractions, so the use of manipulatives may be counterproductive in some cases. When teachers lack the conceptual knowledge to facilitate students' exploration and inquiry, they may be unable to use hands-on tools to help students learn novel material (Bobos, 2017). Thus, manipulatives are a sound tool for the mathematics classroom, but only when the teacher utilizes them correctly, which requires conceptual understanding and may need to be taught explicitly.

Effective Instructional Strategies. While future teachers face numerous obstacles to fraction mastery, a review of the literature revealed approaches that improved knowledge, skills, and understanding among PSTs. All 11 quasi-experimental studies reported improvement in fraction knowledge and skills, but the two most promising approaches were using measurement/number lines and emphasizing conceptual understanding. Participants in Harvey's (2012) study, using an elastic strip as a number line, showed improvement in their ability to identify misconceptions, understand equivalent fractions, order fractions, and use benchmarking to compare fractions. Whiteacre and Nickerson (2016) used a number line intervention with 29 PSTs, and the number of solution strategies used by participants increased by 58% (from 4.86 to 7.7), and correct responses increased by 29% from 5.86 to 7.57. The third number line study by Bobos (2017) showed a greater understanding of fractions among 38 participants. However, the intervention took place over 8 weeks, which may not be feasible for 14-15 week courses. Collectively, these findings show that using measurement and number lines may help PSTs improve their overall mathematics skills and their ability to teach effectively. Furthermore, number lines are suggested as an effective strategy by the National Council of Teachers of Mathematics (NCTM), and most states include number lines as a method of teaching fractions in their standards (NCTM, 2021). Siegler et al. (2011) also shared that behavioral and brain research supports the idea that a number line will help with learning fractions, especially since fraction magnitude and arithmetic ability are closely linked (Siegler et al., 2011). With a better understanding of fraction magnitude, future teachers will also possess more strategies for guiding all students in their classrooms.

Another approach with promising outcomes for fraction mastery is placing a greater emphasis on teaching conceptually. Many errors with rational numbers may not be due to computation mistakes but misapplied or misunderstood concepts (Cortina et al., 2014). Studies that focused on concepts showed improvement among PSTs in a variety of ways. Participants in Reeder and Utley's (2017) study improved their ability to define and order fractions by 20% (46% correct pretest). Interestingly, the PSTs in the study had all taken at least 12 credit hours of mathematics courses before the study, but many still possessed limited knowledge of fractions. In another study, Sahin et al. (2020) increased the number of PSTs who could correctly solve and label fraction problems significantly. On the pretest, 26% of PSTs solved their problems correctly, while two participants could label their images correctly. After the intervention, 60% of PSTs had correct answers, and 20 had the proper label. Finally, Ilyas et al. (2014) used a six-day professional development program that focused on conceptual understanding through learner-centered constructivist activities. The approach allowed participants more time to think, question, and discuss concepts they explored. The mean score from pre- to post-test increased from 10.73 to 17.87, though data on statistical significance was not included, which would need to be addressed in future studies. These studies indicate PSTs need to understand concepts better which can lead to more accurate strategy choices, a better ability to identify student errors, etc. Consequently, this will also benefit the students they teach (Basturk, 2016; Fazio et al., 2016).

Findings from Exploratory Studies. Although the quasi-experimental studies provided insight into PSTs' fraction skills, the exploratory studies reviewed also illuminated factors affecting work with future educators. This includes highlighting the prevalence of issues, which errors were most common, and how deficiencies can affect students' performance. For example, Lee (2017) reported that only 47% of PSTs were able to provide correct answers to fraction problems (30% conceptual errors and 23% algorithmic mistakes). Furthermore, of 85 participants, only two (2.4%) said that they did not have difficulties when working with fractions (Basturk, 2016). In another recent study, only six of 69 PSTs could correctly identify 12 pictorial examples and non-examples of 1/4 (Lee & Lee, 2020). Of greatest concern was that some PSTs believed one could add and subtract fractions with different denominators/referent wholes (Baek et al., 2017), which is a fundamental conceptual error that will cause numerous issues moving forward. One potential reason for the prominence of these errors is relying on the area model too often, which leads to difficulty in explaining algorithms (Alenazi, 2015). The author posits that the area model issue is exacerbated by students being rushed through fractions in elementary school. Even by the time one reaches college, students who struggled with fractions commonly said they only knew one representation as that was the way they were taught (Basturk, 2016). These factors combined may help to explain why PSTs choose poor strategies when working with fractions, which can lead to errors when solving problems. While it is promising that 98% of PSTs were able to pick strategies that might work, only 56% chose the most effective strategy (Thanheiser et al., 2016). In addition, Baek et al., (2017) found that 19% of PSTs used a method or model that was incorrect or invalid, and 83% of those participants chose methods based on misconceptions. PSTs who do not understand concepts will make errors because of their misconceptions, but the fact that strategy choice suffers also highlights that their ability to teach others may suffer.

Since underlying concepts and strategies for understanding and solving problems are so important to growth and learning, the weaknesses some PSTs possess may negatively affect their students' performance. To develop a deep understanding of mathematics, using non-examples helps significantly, but as noted above, Lee and Lee (2020) showed most PSTs are unable to do this effectively. When teachers cannot identify non-examples themselves, they will also struggle with connecting symbolic representations with the real world, which is another strategy that makes fractions more understandable (Lee, 2017). Furthermore, teachers who do not understand why finding a common denominator is necessary will struggle to help their students learn to add and subtract fractions (Baek, 2017). Another issue arising from PSTs' errors is that many could only identify students' procedural errors but could not identify conceptual shortcomings (Basturk, 2016). Since many students will make conceptual errors along the way, PSTs need to be able to spot them, but unfortunately, some may mark a student's answer incorrect even when done properly (Lee & Lee, 2020). This adds to the frustration and confusion students feel when working with fractions. Therefore, opportunities exist for EPPs to modify PST mathematics courses to explore more representations and delve into conceptual discussions. Since math is a continuous content area and builds on concepts, failing to correct conceptual errors will compound as students advance through additional courses. While some teachers may be able to overcome these issues, taken collectively, the literature highlights the need to find ways to help more PSTs improve their fraction knowledge, so their future students are not negatively affected.

Discussion

Although the review only consists of 19 studies, they reveal guidance for EPPs and potential areas for future research. Since students' scores in mathematics have remained virtually stagnant (i.e., no statistically significant differences) since 2003, and many teachers still struggle to teach fractions effectively, what many are doing now may not be particularly effective (OECD, 2018). Since fractions also have myriad real-world applications and set the foundation for concepts like probability and algebra, we must help PSTs improve their fraction knowledge and pedagogical skills before they reach the classroom (Bruce et al., 2013). When one looks at the literature collectively, potential research projects and implications for EPPs emerge. If we can improve PSTs' fraction knowledge and ability to teach a diverse group of students through further research and the approaches EPPs take, we may eventually see the trickle-down effect of improved test scores.

Implications for Educator Preparation Programs. Students who are sitting in teacher preparation classes now have all taken multiple math courses and have passed numerous mathematics tests along the way, but their knowledge of fractions may prevent them from teaching effectively. They have also been in classrooms over the years where their teachers employed copious practice, worksheets, quizzes, etc., but they still struggle with conceptual knowledge (Gabriel, 2016). Although passing exams and reaching the third year of college should be a sign that a person has learned the basics of mathematics, this may not be the case. Therefore, EPPs should consider assessing PSTs' fraction knowledge when they begin their program, especially as it pertains to conceptual understanding. Since only 49% of 8th graders can correctly put fractions in order, and problems tend to persist over time, PSTs may not enter their EPP with the mathematics knowledge they need (Roesslein & Codding, 2019). If these areas do not improve for PSTs, they may transfer these issues to future cohorts of children, who will then also struggle with fractions. Consequently, these students will have challenges when they encounter probability and algebra and may have heightened levels of mathematics anxiety (Bruce et al., 2013; Fazio et al., 2016; Gabriel, 2016). These factors and the literature's support of initial assessment due to the diversity of PSTs indicate that EPPs and their students would benefit by providing the data needed to target instruction for those who may need it.

When testing has revealed where PSTs stand, EPPs should focus their instruction on the areas of greatest need like comparing and ordering fractions, division, choosing proper strategies, and explaining concepts. Interestingly, aside from pedagogical knowledge, the challenges PSTs have mirror those of their students, which emphasizes the need to ensure programs address PSTs' weaknesses before they enter the classroom. As mentioned previously, PSTs struggle with ordering fractions, magnitude, etc. If one looks at the areas of concern among K-12 students, it is logical to think that PSTs' areas of weakness are linked to students' performance. For example, Fazio et al. (2016) found 49% of students who took the NAEP could not correctly order fractions between 0 and 1. Magnitude, which PSTs struggled with, was closely linked to one's arithmetic ability and helps in remembering the steps to solve problems (Siegler et al., 2011). Furthermore, students who did not perform well seemed to lack an understanding of magnitude and used whole-number reasoning as they looked at the numerator and denominator as individual numbers when comparing fractions (Gomez & Dartnell, 2019; Mendiburo et al., 2014). The information above shows that PSTs and current K-12 students are facing the same challenges, which need to be addressed so the cycle does not continue, and EPPs can play a role in improving these issues if they are willing to challenge the status quo.

Traditional school settings teach students to implement procedures and rules, but teachers must know a variety of representations and solution strategies coupled with strong conceptual knowledge to experience the most success (Crawford, 1996). When one knows procedures but not concepts, their ability to differentiate instruction to meet the needs of all students is hindered. Furthermore, since numerous PSTs have issues with choosing correct strategies in their own mathematics work, they may not be able to select developmentally appropriate methods when teaching. However, when PSTs possess multiple solution strategies that are correctly applied, they can better differentiate instruction and guide more students to success. Therefore, EPPs must ensure that courses for aspiring teachers include various fraction representations and discuss alternative solution strategies. If we continue on the current path with the sole use of the area model, we will continue to see problematic data. Zhang et al. (2014) found that when using the area model alone, more than 30% of students failed to give correct answers when dealing with real-world problems, number lines, and simplifying fractions. Representations should include those that view rational numbers as part-whole, division, measurement, operator, and ratio (Van de Walle et al., 2019). Since students who relied on steps and rules were outperformed by those who had strong conceptual knowledge on top of procedural knowledge, it is important to equip PSTs with the necessary tools (Pantziara & Philippou, 2011). Thus, EPPs should work to ensure that activities and assessments that can accomplish multiple goals are used. For example, using the number line and measurement provides an additional representation, improves magnitude understanding, and makes it easier to make real-world connections. If EPPs assess PSTs' fraction knowledge, emphasize several representations and solution strategies, and incorporate number lines, future teachers will be better equipped to help children master fractions.

Future Research. Since success with fractions is one of the foremost building blocks to attainment in algebra 1, high school, and career choice, teachers must possess strong fraction knowledge and pedagogical skills (Shin & Bryant, 2015; Siegler et al., 2012). Yet, many of our future teachers struggle with conceptual knowledge, choosing effective strategies for solving problems and understanding that fractions are numbers one can arrange on a number line (magnitude). Therefore, future research should investigate which strategies and approaches are most effective in helping PSTs understand the underlying concepts that guide algorithms and procedures. For example, inquiry can look at strategies that would help educators understand how inverting and multiplying work and why the algorithm is used. In the classes I teach, students are always unaware that dividing straight across is possible, but after showing examples, they can see the utility of inverting and multiplying. However, they are shocked when I share that, in some cases, dividing straight across is not only possible but the easiest manner of solving with less room for error. While the literature includes some methods for discussing concepts, it would benefit the field to understand the best ways to help PSTs understand fundamental fractions concepts.

Interestingly, a potential starting point is to look at elementary student research and apply the strategies that were found to be effective. Since students and PSTs have similar challenges, they may benefit from similar interventions. For example, in a number line study of sixth-grade students, those in the experimental group showed significant gains over the control, and this held even after a seven-week post-test (Barbieri, 2020). A similar approach may help PSTs, so they are better prepared for their classroom. Furthermore, this type of research may help to determine why many teachers only use one fraction representation, so we can help others learn and apply different representations and real-world applications. Primarily using area models may account for some fraction weaknesses and why people could not transfer fraction knowledge to real-world contexts. As Zhang et al. (2014) found, many texts and teachers claim to use a variety of teaching models and methods, but most of them would still be considered the area model. This further shows the weaknesses that exist with concept knowledge, as many are trying new strategies but do not understand fractions well enough to know they are still addressing part-whole depictions.

Finally, research should look at how a teacher's knowledge and skill truly affect student performance. If PSTs and in-service teachers' fraction knowledge is lacking, it is important to understand how this affects their students' performance in mathematics. For example, Clark and Roche (2009) found that people who used benchmarking and residual strategies had the best conceptual understanding, but their teachers were not teaching them in class. Research should investigate why teachers are not using these strategies if we know they are used by those who perform best. In addition, investigations should look at why educators replace effective strategies with things like placing immense emphasis on memorizing procedures and tricks, and how teachers approach explanations of fractions. These investigations will provide more information that may help us move from a formulaic, procedural, tedious classroom to focus more on developing concepts and improving PSTs' overall skills and knowledge.

Conclusion

Although many fraction studies of elementary students exist, the research concerning PSTs' fraction knowledge and skill appears to be limited. Considering the role fractions play in courses like algebra, and the fact that half of eighth-grade students in the United States failed to order

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three fractions, EPPs must work to better equip our PSTs (Gabriel, 2016; Siegler et al., 2011). The literature shows PSTs struggle with similar issues as elementary students, which may be due to teachers scaffolding students when they possess limited knowledge. If PSTs can promote the five strands of mathematical proficiency and offer differentiation through various representations, strategies, etc., they may be able to use the ZPD as intended to improve their students' math performance (Van de Walle, 2019; Denhere et al., 2013). Unfortunately, most people learned in an atmosphere where math was decontextualized from the real world, fractions were rushed through, and the focus was on procedures rather than concepts (Alenazi, 2015; Crawford, 1996). This has led to teachers who fail to provide the proper assistance required, but when the ZPD theory is applied properly, it can improve performance and achievement (Denhere et al., 2013). As Reeder and Utley (2017) stated,

> Although these prospective teachers had many years of school mathematics, including four college-level mathematics courses and one primary-level mathematics methods course, their reasoning about basic fraction concepts was quite shallow and based heavily on misconceptions they had previously developed. (p. 314)

All of the information revealed by the literature shows that EPPs cannot take PSTs' fraction knowledge for granted. PSTs who have misconceptions may pass those on to their students who will develop similar limits to their knowledge. Since misconceptions students develop in third and fourth grade still exist in high school and beyond, it is imperative to address these issues (Siegler et al., 2012). A greater focus on concepts, using more representations, and developing a wider range of solution strategies all appear to move PSTs closer to mastery. By investigating the obstacles to PSTs' mastery and promoting targeted interventions, programs can mitigate weaknesses before novice teachers enter the classroom. In addition, education faculty should consider collaborating with those who teach first- and second-year math classes, as this will allow programs to address concerns with fractions before students begin their teacher preparation program. Since many college students besides PSTs struggle with fractions, these collaborative efforts may yield greater reach and also lead to benefits for students in other programs as well (Fazio et al., 2016; Siegler et al., 2011). With the significant influence that fractions have on future math courses and peoples' widespread difficulties with rational numbers, research must continue to determine the impact these strategies may have on PSTs' future students. In so doing, we may find that the next generation of students can avoid the pitfalls so many have experienced presently and in the past.

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