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SUPERLATIVE QUANTIFIERS AS MODIFIERS OF META-SPEECH ACTS

ABSTRACT: The superlative quantifiers, *at least* and *at most*, are commonly assumed to have the same truth-conditions as the comparative quantifiers *more than* and *fewer than*. However, as Geurts & Nouwen (2007) have demonstrated, this is wrong, and several theories have been proposed to account for them. In this paper we propose that superlative quantifiers are illocutionary operators; specifically, they modify meta-speech acts.

Meta speech-acts are operators that do not express a speech act, but a willingness to make or refrain from making a certain speech act. The classic example is speech act denegation, e.g. *I don't promise to come*, where the speaker is explicitly refraining from performing the speech act of promising. What denegations do is to delimit the future development of conversation, that is, they delimit future admissible speech acts. Hence we call them meta-speech acts. They are not moves in a game, but rather commitments to behave in certain ways in the future. We formalize the notion of meta speech acts as commitment development spaces, which are rooted graphs: The root of the graph describes the commitment development up to the current point in conversation; the continuations from the root describe the admissible future directions.

We define and formalize the meta-speech act GRANT, which indicates that the speaker, while not necessarily subscribing to a proposition, refrains from asserting its negation. We propose that superlative quantifiers are quantifiers over GRANTS. Thus, *Mary petted at least three rabbits* means that the minimal number n such that the speaker GRANTS that Mary petted n rabbits is $n = 3$. In other words, the speaker denies that Mary petted two, one, or no rabbits, but GRANTS that she petted more.

We formalize this interpretation of superlative quantifiers in terms of commitment development spaces, and show how the truth conditions that are derived from it are partly entailed and partly conversationally implicated. We demonstrate how the theory accounts for a wide variety of phenomena regarding the interpretation of superlative quantifiers, their distribution, and the contexts in which they can be embedded.

1. THE MEANING OF SUPERLATIVE QUANTIFIERS

1.1. Commonly held intuitions

What do superlative quantifiers, like *at least* and *at most*, mean? Non-linguists have a clear intuition—for example, note the following discussion of *at least* from a book about computer databases:

One important rule to remember is that there should be at least $(n - 1)$ joins in an n -table query; thus, you need at least two joins for a three-table query, at least three joins for a query that involves four tables, and so on. The words “at least” are important: there could be more than $(n - 1)$ joins... but if your multitable query has less than $(n - 1)$ joins, the result will be [bad] (A. Krieger and B. M. Trukhnov, *SQL Bible*, p. 319).

According to this intuition, *at least* x means x or more, but not less; *at most* x means x or less, but not more. In a context in which only integers are relevant, things are even simpler: *at least* x means greater than $x - 1$, and *at most* x means fewer than $x + 1$.

Thus, since it is impossible to pet a non-integer number of rabbits, (1-a) would mean (1-b).

- (1) a. John petted at least three rabbits.
- b. John petted more than two rabbits.¹

Similarly, (2-a) would be equivalent to (2-b).

- (2) a. John petted at most three rabbits.
b. John petted fewer than four rabbits.

1.2. Keenan and Stavi (1986)

Until recently, such intuitions were widely shared by linguists as well, and have been formalized by Keenan & Stavi (1986). According to their theory, both superlative (*at least*, *at most*) and comparative (*more than*, *fewer than*) quantifiers are treated simply as generalized quantifiers, i.e. relations between sets. Thus, the meaning of (1-a) is simply (3-a), where R is the set of rabbits, and the meaning of (2-a) is simply (3-b).

- (3) a. $|R \cap \lambda x. \text{pet}(j, x)| \geq 3$
b. $|R \cap \lambda x. \text{pet}(j, x)| \leq 3$

Besides being intuitive, this definition has two important advantages. One advantage is that it gets the truth conditions right: if John petted two or fewer rabbits, (1-a) is false, and if he petted four or more rabbits, (2-a) is false.

The second advantage is that these truth conditions are extensional, which is as it should be. For example, suppose Mary likes rabbits but no other animal. Then, the extensions of the predicates *rabbit* and *animal that Mary likes* are the same. Note that the truth values of (1-a) and (4) are the same, indicating that superlative quantifiers are extensional.

- (4) John petted at least three animals that Mary likes.

However, Keenan & Stavi's theory, while getting the truth conditions right, fails to account for a number of phenomena. In particular, as Geurts & Nouwen (2007) point out, the meanings of superlative quantifiers differ from those of comparative quantifiers (*more/fewer than*) in subtle ways.

One of the observations of Geurts & Nouwen is that the distribution of comparative quantifiers is more restricted than that of superlative quantifiers. For example, only the latter, but not the former, can take sentential scope:

- (5) a. John petted three rabbits $\left\{ \begin{array}{l} \text{at most} \\ * \text{fewer than} \end{array} \right\}$.
b. $\left\{ \begin{array}{l} \text{At least} \\ * \text{More than} \end{array} \right\}$, John petted three rabbits.

Additionally, superlative quantifiers, but not comparative ones, may combine with quantifiers, proper names, and specific indefinites:

- (6) a. Mary petted $\left\{ \begin{array}{l} \text{at least} \\ * \text{more than} \end{array} \right\}$ every young rabbit.
b. Mary petted $\left\{ \begin{array}{l} \text{at most} \\ * \text{fewer than} \end{array} \right\}$ Bugs Bunny.
c. John petted $\left\{ \begin{array}{l} \text{at least} \\ * \text{more than} \end{array} \right\}$ two rabbits, namely Bugs Bunny and Peter.

There are, however, cases where comparative quantifiers are acceptable, and it is superlative quantifiers that are odd. Suppose John petted exactly three rabbits, and we know this. Based on this fact, we would be justified in uttering (1-b); however, it would be quite strange to utter (1-a) or (2-a).²

There are differences between superlative and comparative quantifiers not just in distribution, but also in interpretation: the former lack some readings that the latter have. For example, (7-a) is ambiguous: it can mean either that it is permissible for you to have fewer than three martinis (say, because you don't like martinis), or that you may not have more than two martinis. In contrast, (7-b) is not ambiguous, and only receives the second reading.

- (7) a. You may have fewer than three martinis.
b. You may have at most two martinis.

1.3. Geurts and Nouwen (2007)

In order to account for these phenomena, Geurts & Nouwen (2007) argue against the commonly held intuition, and propose that comparative and superlative quantifiers have different interpretations.

Following Krifka (1999a), they propose that comparative quantifiers are focus sensitive NP modifiers. Roughly, (1-b) means that there

is a property that is higher or equal on the relevant scale than the property of petting two rabbits, and this property applies to John.³

Formally, the meaning Geurts & Nouwen propose for *more than α* is:

$$(8) \quad \lambda x. \exists \beta (\beta > \alpha \wedge \beta(x))$$

The relevant scale is affected by focus, which explains the difference between (9-a) and (9-b).

- (9) a. John petted more than [two]_F rabbits.
b. John petted more than [two rabbits]_F.

Sentence (9-a), with focus on *two*, means that John petted a number of rabbits, and this number is greater than two; while (9-b), with focus on *two rabbits*, is compatible with John having petted exactly two rabbits, provided that he petted additional animals.

Sentences (9-a) and (9-b) are evaluated with respect to different scales; and yet other scales account for examples such as the following:

- (10) a. I will be more than happy to send you the necessary forms.
b. The telephone service here is less than satisfactory.

Sentence (10-a) is presumably evaluated with respect to a scale involving properties such as being reluctant, indifferent, happy and ecstatic; (10-b) presumably involves properties such as being terrible, bad, satisfactory, good, and excellent.

Importantly, Geurts & Nouwen restrict α and β in (8) to denote only first-order properties, i.e. expressions of type $\langle e, t \rangle$. The properties *happy* and *satisfactory* are clearly first-order. The property of being a group of two rabbits is also first-order in their system, since they treat groups as individuals. But propositions are not first-order properties, which is why comparative quantifiers cannot combine with them, and the unacceptability of the sentences in (5) is thereby explained. Similarly, quantifiers, names, and specific indefinites also do not denote first-order properties, which is why the sentences in (6) are bad.

Regarding superlative quantifiers, Geurts & Nouwen propose that they are epistemic operators. Specifically, the meanings of (1-a) and (2-a) can be roughly paraphrased as (11-a) and (11-b), respectively.

- (11) a. It is epistemically necessary that John petted three rabbits, and it is epistemically possible that he petted more.
b. It is epistemically possible that John petted three rabbits but it is epistemically impossible that he petted more.

Geurts & Nouwen demonstrate how their approach solves some of the problems with Keenan & Stavi's theory. In particular, they can explain why (2-a) would be odd if it is known that John petted exactly three rabbits: according to Geurts & Nouwen's theory, (2-a) entails that it is epistemically possible that John petted three rabbits. But in this case it is not only epistemically *possible*, but, in fact, epistemically *necessary* that John petted three rabbits, so the speaker makes a weaker statement than the one she can and ought to make. In other words, saying that it is epistemically possible that John petted three rabbits implicates that it is not epistemically necessary; but this implicature is not satisfied in a situation where it is known that John petted three rabbits.

According to Geurts & Nouwen (1-a) is not merely odd, but actually false: this is because it entails that it is epistemically possible that John may have petted more than three rabbits; but if it is known that he petted exactly three, this entailment is plainly false.

Crucially, superlative quantifiers are not restricted to combine only with first-order properties: hence, their distribution is less restricted than that of comparative quantifiers, and the facts exemplified by the sentences in (5) and (6) are thereby explained.

The missing reading of (7-b) is explained by the analysis of superlative quantifiers as epistemic modals: in general, deontic modals cannot take scope over epistemic ones.

Thus, Geurts & Nouwen's theory successfully accounts for a number of puzzling phenomena. However, they make these gains at a considerable cost. One problem is that the superlative morphology of superlative quantifiers is ignored: it is not reflected in the analysis. There is nothing in the logical form proposed by Geurts & Nouwen for (1-a) that indicates that three is the *least* number of rabbits that John petted.

Moreover, Geurts & Nouwen lose the two major advantages of Keenan & Stavi's theory: correct truth conditions, and extensionality.

Basing the truth conditions of superlative quantifiers on epistemic modality makes them subjective, depending on epistemic states, which leads to incorrect predictions. Suppose John petted exactly four rabbits

in the actual world; then (1-a) would be true and (2-a) would be false, regardless of any belief worlds.

One may try to save Geurts & Nouwen's theory by treating the epistemic modals as objective. Lyons (1977) draws a distinction between *subjective* and *objective* senses of epistemic modals. Thus, (12-a) is subjective, depending on the epistemic state of the speaker; but (12-b) is "objective", in the sense that it depends on the epistemic states of a large number of people, who are collectively the authority on the subject.

- (12) a. Has anybody heard the news? I want to know who won the match. It might have been Mark.
 b. There might have been life on Mars at some point in the past.

However, treating Geurts & Nouwen's epistemic modals as objective would not help. Suppose John committed exactly four traffic violations, but nobody knows this, not even the police (who are the authority on the subject), and not even John himself. Then, it would still be true that he committed at least three traffic violations, and false that he committed at most three traffic violations, and these truth values depend only on what actually happened, not on anybody's beliefs.

The analysis of superlative quantifiers as epistemic modals also leads to the incorrect prediction that they are intensional. However, as we have seen above, superlative modifiers are extensional. Consider again the case where Mary likes rabbits, but no other animal. But this fact may not be known; hence, it may be epistemically necessary for John to pet three rabbits, without it being epistemically necessary for him to pet three of the animals that Mary likes. Hence, it is predicted that (1-a) may have a different truth value from (4), repeated below:

- (13) John petted at least three animals that Mary likes.

But this prediction is wrong.

At most licenses negative polarity items, whereas *at least* does not:

- (14) At $\left\{ \begin{array}{l} \textit{most} \\ \textit{*least} \end{array} \right\}$ three people have ever been in this cave (last century).⁴

However, nothing in Geurts & Nouwen's theory predicts this behavior.

Geurts et al. (2010) present the results of experiments that are claimed to support this theory; but some of the results actually are in conflict with it. Recall that, according to this theory, there is a fundamental difference between *at least* and *at most*: if John petted exactly three rabbits and the speaker knows this, (1-a) would be false, whereas (2-a) would be true but infelicitous. As a consequence, (1-a) is predicted to be ruled out in this situation; however, citing Noveck (2001) and their own unpublished study, Geurts et al. conclude that true but infelicitous sentences such as (2-a) ought to receive a mixed response, i.e. be acceptable about half the time.

Geurts et al.'s prediction, however, is not borne out. They asked subjects to judge whether (15-b) and (15-c) follow from (15-a).

- (15) a. Wilma had three beers.
 b. Wilma had at least three beers.
 c. Wilma had at most three beers.

The results were that *both* (15-b) and (15-c) are accepted about half the time.

Geurts et al. are aware of this difficulty, and they attempt to explain it by hypothesizing that people who accept (15-b) do so because they interpret (15-a) as saying that Wilma had three beers or more. As support for this claim, they demonstrate that when (15-a) is replaced with (16), the acceptance of the inference to (15-b) is reduced significantly.

- (16) Wilma had exactly three beers.

However, this argument is not convincing, for two reasons. One is that the acceptance rate is about 20%, still significantly higher than the predicted 0%. The second reason is that there is a much simpler explanation for why *exactly* reduces the acceptance of the inference: plausibly, the word *exactly* itself acts as a superlative quantifier. Evidence for this possibility comes from the fact that *exactly* cannot co-occur with superlative quantifiers:

- (17) *John petted $\left\{ \begin{array}{l} \textit{at least} \\ \textit{at most} \end{array} \right\}$ exactly three rabbits.

Therefore, an utterance of (16) introduces a set of alternative sentences involving other superlative quantifiers— (15-b) and (15-c)— and implicates that these alternatives are false.

In this paper, we propose that superlative quantifiers are illocutionary operators. To make this view explicit, we need to look more closely at speech acts.

2. MODELING SPEECH ACTS AND META-SPEECH ACTS

2.1. Commitment developments

We understand speech acts as changing commitments of the interlocutors. A conversation will consist of a sequence of such commitment changes, which we will call a **commitment development**. We model commitment developments as follows. Let us assume that c is the current commitment, a commitment development which incorporates the changes that were enacted up to this point. Let us assume that $A(s,a)$ is a speech act by speaker s to addressee a , which results in the obligations $\text{obl}(A(s,a))$ (where it is irrelevant, we will suppress speaker and addressee arguments, and just write A). Then the enactment of $A(s,a)$ at the commitment development c can be modelled as follows:

$$(18) \quad c + A = \{c, \text{obl}(A)\}$$

As a result, we have a set consisting of the input commitment development c and the new commitments, which is the output commitment development. Note that c represents the development so far, and is therefore a member of the output, as the output “memorizes” the way that led to this state.

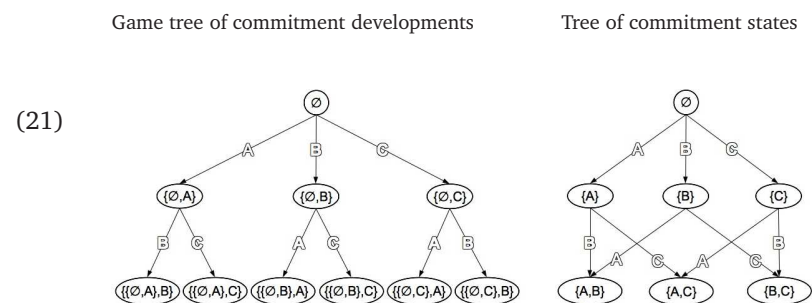
We represent the initial commitment development by the empty set, indicating that nothing has happened so far. The added commitment or commitments are not themselves sets but commitments specified in some representation language. For example (where Φ is a proposition and $\underline{\Phi}$ a set of propositions):

$$(19) \quad \begin{aligned} c + \text{ASSERT}(s,a)(\Phi) &= \{c, s \text{ guarantees } a \text{ that } \Phi \text{ is true}\} \\ c + \text{QUEST}(s,a)(\underline{\Phi}) &= \{c, s \text{ obliges } a \text{ to assert those} \\ &\quad \text{propositions } p \text{ in } \underline{\Phi} \text{ that } a \text{ can} \\ &\quad \text{truthfully assert}\} \end{aligned}$$

For illustration, consider a case in which a commitment development c undergoes two subsequent changes:

$$(20) \quad c + A + B = \{c, \text{obl}(A)\} + B = \{\{c, \text{obl}(A)\}, \text{obl}(B)\}$$

The resulting structure is a tree. For example, with the empty context as a root and three speech acts A, B, C , and ruling out repeated enactments of the same speech acts, we have the commitment developments for the first two steps as depicted on the left-hand side of the following diagram (where we write $\{c, A\}$ for $\{c, \text{obl}(A)\}$, for short).



This is a so-called game tree that shows the possible courses that a game might take. We write $c < c'$ iff c' is a commitment continuation that succeeds c . This notion can be defined recursively as follows:

$$(22) \quad \begin{aligned} \text{a. } &c < \{c, \text{commitments}\} \\ \text{b. } &\text{if } c < c' \text{ and } c' < c'', \text{ then } c < c'' \end{aligned}$$

Update of commitment developments is order sensitive, that is, $c + A + B$ is different from $c + B + A$. Sometimes we do not care about the order in which the commitments have been introduced. We can get rid of the order information by recursive union, \cup :

$$(23) \quad \begin{aligned} \text{a. } &\cup\{\emptyset, o_1, \dots, o_n\} = \{o_1, \dots, o_n\}, \text{ where } o_1, \dots, o_n: \text{ obligations.} \\ \text{b. } &\cup\{c, o_1, \dots, o_n\} = \{\cup c, o_1, \dots, o_n\} \end{aligned}$$

For example,

$$(24) \quad \cup\{\{\emptyset, \text{obl}(A)\}, \text{obl}(B)\} = \{\text{obl}(A), \text{obl}(B)\}$$

We call such unordered sets of commitments **commitment states**. Commitment states lead to a more compact representation of language games than commitment developments. For example, the game tree on the left-hand side of (21) results in the representation specified on the right-hand side, using commitment states.

As in other games, not every transition is a possible move. One important rule is that the update of commitment developments comes with a requirement that new commitments do not contradict existing ones. For example, if a speaker has asserted Φ , this speaker cannot assert $\neg\Phi$ later:

$$(25) \quad c + \text{ASSERT}(s, a)(\Phi) = \{c + s \text{ guarantees } a \text{ that } \Phi \text{ is true}\}, \\ \text{provided that} \\ [s \text{ guarantees } a \text{ that } \neg\Phi \text{ is true}] \notin \cup c$$

This is certainly not the case in everyday conversation; people do change their minds. But here we restrict ourselves to the basic case of monotonic update.

The notion of updates of commitment developments allows for a straightforward treatment of conjunction of speech acts. As a matter of fact, there are two types of conjunction: dynamic conjunction ‘;’ and static conjunction ‘&’:

$$(26) \quad \text{a. } c + [A ; B] = c + A + B \\ \text{b. } c + [A \& B] = [c + A] \cup [c + B] = \{c, \text{obl}(A), \text{obl}(B)\}$$

These two notions of speech act conjunction coincide if we model conversation with commitment states instead of commitment developments.

$$(27) \quad \cup [c + [A ; B]] = \cup [c + [A \& B]] = \cup [c + A] \cup \cup [c + B]$$

Krifka (1999b, 2001) has argued that speech act conjunction is needed to deal with quantification into questions, as in the following example, which has a reading under which the universal quantifier appears to scope out of the speech act (cf. Karttunen 1977):

$$(28) \quad \text{a. } \text{How many rabbits did every child pet?}$$

- b. $\forall x[\text{child}(x) \rightarrow \text{How many rabbits did } x \text{ pet?}]$
 c. $= \forall x[\text{child}(x) \rightarrow \text{What is the number } n \text{ such that } x \text{ petted } n \text{ rabbits?}]$

The formula illustrates the intended analysis only but cannot be taken literally, as its consequent is a question, not an expression that evaluates to a truth value. But as universal quantifiers are generalized conjunctions, we can assume that the whole expression denotes a conjoined speech act:

$$(29) \quad c + [\text{How many rabbits did every child pet?}] \\ = c + \&_{x \in \text{child}} \text{QUEST}(\text{How many rabbits did } x \text{ pet}) \\ = c + [\text{QUEST}(\text{How many rabbits did John pet}) \& \\ \text{QUEST}(\text{How many rabbits did Mary pet}) \& \\ \text{QUEST}(\text{How many rabbits did Sam pet}) \dots]$$

This adds to the commitment development c the commitments that come with the indicated questions; that is, if accepted, the addressee has to give answers to these questions.

The analysis of quantification into questions (and other speech acts) works because it involves universal quantification, which can be reduced to conjunction. Other quantifiers, like *most*, do not allow for this reading:

- $$(30) \quad \text{a. } \text{How many rabbits did most children pet?} \\ \text{b. } = \text{What is the number } n \text{ such that most children petted } n \text{ rabbits?} \\ \text{c. } \neq \text{For most children } x: \text{How many rabbits did } x \text{ pet?}$$

This is because *most* cannot be reduced to conjunction, but requires disjunction for its definition, and disjunction is not defined for speech acts in general. To be sure, we do find disjointed questions, but they are not interpreted as expressing an obligation on the addressee that either the first or the second question should be answered. Depending on intonation, they get other interpretations, e.g. as alternative questions. For example, the following question, with rising accent on *rabbit* and falling accent on *chameleon*, is an alternative question that presupposes

that either Mary petted a rabbit or that John poked at a chameleon, and the addressee should specify the proposition that is true.⁵

(31) Did Mary pet a rabbit, or did John poke at a chameleon?

If a commitment development could be updated with a disjunction of two speech acts, the result would be an ambiguous commitment development. Such ambiguities would quickly multiply if additional disjunctive speech acts were uttered. This is a processing constraint for speech acts. In the next section, we will define the notion of commitment development spaces, which exclude speech act disjunction in principle, except in special cases where they do not lead to ambiguous commitment developments.

2.2. Commitment development spaces: Denegation

The notion of a commitment development has to be broadened to deal with so-called speech act denegation, as in the following example (cf. Searle 1969):

(32) I don't promise to come.

According to Hare (1970), the speaker is “explicitly refraining from performing the speech act in question”. This cannot be expressed as an update of a commitment development, as a regular speech act. Also, notice that denegations cannot be marked with *hereby*:

(33) *I hereby don't promise to come.

What denegations do is to delimit the future development of conversation, that is, they delimit future admissible speech acts. Hence we call them **meta-speech acts**. They are not moves in a game, but rather commitments to act in certain ways in the future. This requires a more general setup for the modelling of conversational games.

In this more general setup, we introduce the notion of **commitment (development) spaces** as a **rooted set** of commitment developments. Formally, a set C of commitment developments is a commitment space if the following holds:

(34) There is a $c \in C$ such that for all $c' \in C$: $c \leq c'$

We call the minimal commitment development the **root** of the commitment space, and write \sqrt{C} . For any sentence in discourse, there is a commitment space that defines the current commitments of the discourse participants, which are modelled by a commitment space C . The root of C describes the commitment development up to the current point in conversation; the continuations from the root in C describe the admissible future directions that the commitment might take.

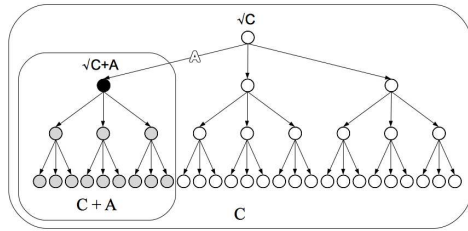
We can think of the commitment space as representing the rules of the conversational game. There are general rules, like the one mentioned above that contradictory commitments should be avoided. But there are also rules that are introduced during the conversation itself. Speech act denegation is such a rule; with *I don't promise to come*, the speaker excludes the move *I promise to come*, at least for the time being.

The enactment of a regular speech act relative to a commitment space, that is, the update of a commitment space C by a regular speech act A , can now be defined as the subset of commitment developments that we get by updating the root of the commitment space:

(35) $C + A = \{c \in C \mid \sqrt{C} + A \leq c\}$

The update of a commitment space C with a speech act A results in the set of commitment developments containing the root of C updated with the speech act A , and all commitment developments of C that continue this updated root. Notice that the definition of update of a commitment space C by a speech act A requires that the set C is a rooted set of commitment developments, as it refers to the root \sqrt{C} , and will lead to another rooted set of commitment developments, the new root being $\sqrt{C} + A$. In the following diagram, the root of the output commitment state is rendered black, and the other common ground developments of the output state are rendered gray.

(36)



The denegation of a speech act also consists in a change of the input commitment space, namely in explicitly refraining from making the speech act that is denegated. We can understand this in one of two ways: Either locally, the speaker refrains from making the speech act at the current point, or globally, the speaker refrains from making the speech act now or at future states. We assume here that denegation has a local character. This is because we find (37-a) to be consistent, in contrast with (37-b).

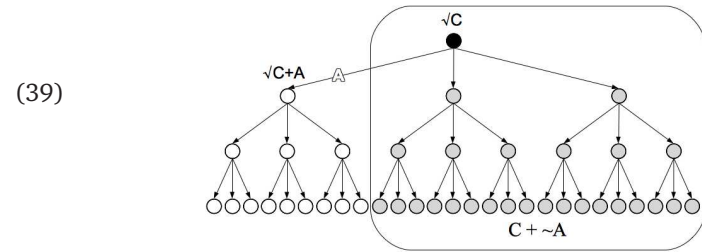
- (37) a. I don't promise to come to your party, but I might promise to later, when I've had a look in my calendar.
- b. # I promise to come to your party, but I might say that I won't come after I've had a look in my calendar.

But we note that speech act denegations are made with the idea that a future enactment of the denegated speech is dependent on some condition, like a change of the state of the world or the available information, as suggested in (37-a) by *when I've had a look in my calendar*.

The weak denegation of a speech act A, for which we will write $\sim A$, then can be stated as follows:

$$(38) \quad C + \sim A = \overline{C + A}$$

This is the complement of the commitment space C updated with A. The result is a rooted commitment state, with the same root as the input state C. Denegation just prunes the tree of admissible developments:



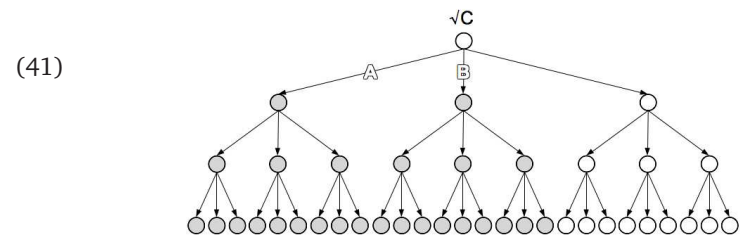
We have the complement rule: $\sim\sim A = A$, as for every C it holds that $\overline{\overline{C + A}} = C + A$.

Dynamic and static conjunction of two speech acts can be modelled as follows:

- (40) a. $C + [A ; B] = \{c \in C \mid \sqrt{C} + [A ; B] \leq c\} = C + A + B$
- b. $C + [A \& B] = \{c \in C \mid \sqrt{C} + [A \& B] \leq c\}$

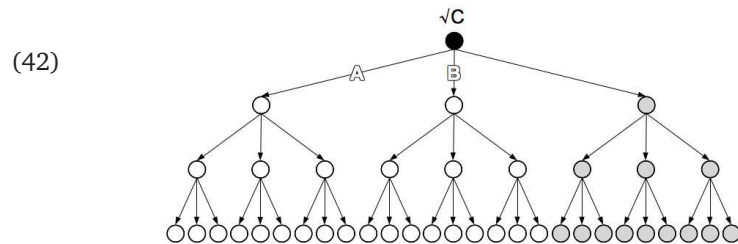
As C is a set (of commitment developments), we now might want to define the notion of a disjunction of speech acts, as set union. But this does not result in a proper commitment space, as the resulting set typically is not rooted.

$$C + [A \vee B] = [C + A] \cup [C + B]$$



If speech acts are transitions from commitment spaces to commitment spaces, then it follows that disjunctions are not defined for speech acts, as the output would not be a commitment state, but just an unrooted set of commitment developments. However, it turns out that the **denegation** of a disjunction of commitment spaces is well defined, as it results in a proper, rooted commitment space:

$$\begin{aligned} C + \sim [A \vee B] &= \overline{[C + [A \vee B]]} \\ &= \overline{[[C + A] \cup [C + B]]} \\ &= \overline{[C + A]} \cap \overline{[C + B]} \\ &= [C + \sim A] \cap [C + \sim B] \end{aligned}$$



This application of de Morgan on the level of speech acts predicts the following equivalence:

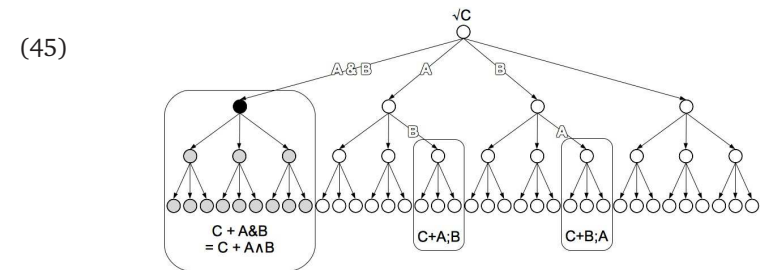
- (43) a. I don't promise to marry you or swear to stay with you
 b. \Leftrightarrow I don't promise to marry you and I don't swear to stay with you.⁶

The operation of intersection in the last line is a kind of Boolean conjunction: it identifies the commitment space that results from C when both the update $\sim A$ and the update $\sim B$ are satisfied. But notice that with simple speech acts A, B, intersection of $C + A$ with $C + B$ typically results in the empty set, as $C + A = \{c \in C \mid \{\sqrt{C}, \text{obl}(A)\} \leq c\}$ and $C + B = \{c \in C \mid \{\sqrt{C}, \text{obl}(B)\} \leq c\}$, which are non-overlapping sets if $\text{obl}(A) \neq \text{obl}(B)$. However, notice that $C + [A \& B] = \{c \in C \mid \{\sqrt{C}, \text{obl}(A), \text{obl}(B)\} \leq c\}$ describes the update of C to a commitment state where the obligations of both A and B are satisfied. Hence we can

define a “Boolean” conjunction of commitment states as follows:

- (44) a. If A & B is defined: $C + [A \& B] = C + [A \& B]$
 b. else, $C + [A \& B] = [C + A] \cap [C + B]$

The case of (44-b), where A & B is undefined, was illustrated above. This definition has the consequence that de Morgan's rule sometimes holds. For the case of (42), the part (44-b) applies: We have $\sim [A \vee B] = [\sim A \& \sim B]$, as $\sim A \& \sim B$ is not defined if A, B are simple speech acts. A case in which (44-a) applies is illustrated below.

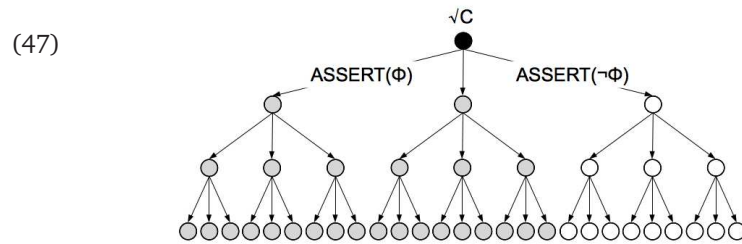


The main use of denegation in this paper is to define the meta-speech act of a GRANT.⁷ A GRANT indicates a willingness to go along with a possible assertion of a proposition by the opponent.

Hence, a GRANT is a denegation to assert the negation of that proposition:

- (46) $\text{GRANT}(\Phi) := \sim \text{ASSERT}(\neg \Phi)$

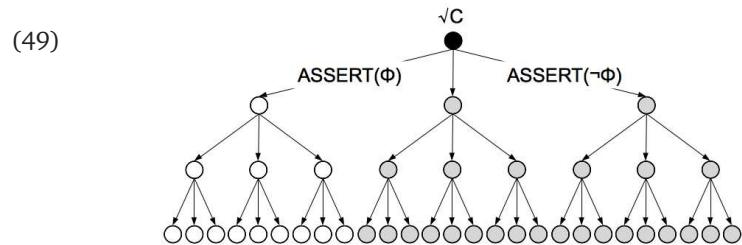
The result of GRANTing a proposition Φ is illustrated in the following diagram. Notice that GRANTs, as denegations, do not change the root. The GRANT includes, but does not enforce, the assertion of Φ .



We have the following equivalence, familiar from the modal logic equivalences $\Box\Phi = \neg\Diamond\neg\Phi$:

$$(48) \quad \text{ASSERT}(\Phi) = \sim \text{GRANT}(\neg\Phi)$$

The following diagram illustrates $\text{GRANT}(\neg\Phi)$; notice that the complement is $\text{ASSERT}(\Phi)$.



3. THE PRAGMATICS AND SEMANTICS OF SUPERLATIVE QUANTIFIERS

3.1. Modelling superlative operators

We are now in a position to model *at least* and *at most* as speech-act related operators, or rather, as operations that help to express meta-speech acts. Take the following example:

$$(50) \quad \text{Mary petted at most three rabbits.}$$

Intuitively, this says that the maximal number n such that the speaker GRANTS that Mary petted n rabbits is $n = 3$.

$$(51) \quad \max n : \text{GRANT}(|\mathbf{rabbit} \cap \lambda x.\mathbf{pet}(m,x)|) = 3$$

This explains the superlative morphology of *at most*. In our current setup, (51) cannot be interpreted directly. But notice that we can interpret it as saying that for all numbers n with $n > 3$, the speaker does not GRANT that Mary petted n rabbits, which is a conjunction of a denial of GRANTS:

$$(52) \quad C + \bigwedge_{n>3} \sim \text{GRANT}(|\mathbf{rabbit} \cap \lambda x.\mathbf{pet}(m,x)| = n)$$

This generalized conjunction is tantamount to:

$$(53) \quad C + \left[\begin{array}{l} \sim \text{GRANT}(|\mathbf{rabbit} \cap \lambda x.\mathbf{pet}(m,x)| = 4) \wedge \\ \sim \text{GRANT}(|\mathbf{rabbit} \cap \lambda x.\mathbf{pet}(m,x)| = 5) \wedge \\ \dots \end{array} \right]$$

With *at most*, the speaker does not make an assertion but rather excludes assertions—here, the assertions that Mary petted 4 or more rabbits. Notice that with (44) and (48), (52) is equivalent to the following assertion:

$$(54) \quad \begin{aligned} & C + \bigwedge_{n>3} \text{ASSERT}(|\mathbf{rabbit} \cap \lambda x.\mathbf{pet}(m,x)| \neq n) \\ & = C + \&\text{ASSERT}(|\mathbf{rabbit} \cap \lambda x.\mathbf{pet}(m,x)| \neq n) \end{aligned}$$

The speaker excludes that Mary petted four or more rabbits, but leaves it open whether she petted three, two, one, or no rabbit at all.

We now consider the case of *at least*, with the following example:

$$(55) \quad \text{Mary petted at least three rabbits.}$$

This says that the *minimal* number n such that the speaker GRANTS that Mary petted n rabbits is $n = 3$:

$$(56) \quad \min n : \text{GRANT}(|\mathbf{rabbit} \cap \lambda x.\mathbf{pet}(m,x)|) = 3$$

This translates into the denegation of GRANTs that Mary petted n rabbits for n smaller than 3:

$$(57) \quad C + \bigwedge_{n < 3} \sim \text{GRANT}(|\mathit{rabbit} \cap \lambda x.\mathit{pet}(m, x)| = n)$$

Which is tantamount to:

$$(58) \quad C + \left[\begin{array}{l} \sim \text{GRANT}(|\mathit{rabbit} \cap \lambda x.\mathit{pet}(m, x)| = 2) \wedge \\ \sim \text{GRANT}(|\mathit{rabbit} \cap \lambda x.\mathit{pet}(m, x)| = 1) \wedge \\ \sim \text{GRANT}(|\mathit{rabbit} \cap \lambda x.\mathit{pet}(m, x)| = 0) \end{array} \right]$$

With *at least*, the speaker excludes assertions that Mary petted 2, 1, or 0 rabbits. This is equivalent to the following assertion:

$$(59) \quad \begin{aligned} & C + \bigwedge_{n < 3} \text{ASSERT}(|\mathit{rabbit} \cap \lambda x.\mathit{pet}(m, x)| \neq n) \\ & = C + \bigwedge_{n < 3} \& \text{ASSERT}(|\mathit{rabbit} \cap \lambda x.\mathit{pet}(m, x)| \neq n) \end{aligned}$$

The speaker excludes that the number of rabbits Mary petted is smaller than 3.

3.2. Truth conditions

Representing superlative quantifiers as modifiers of meta-speech acts raises an immediate question. Recall that we hailed as one of the advantages of Keenan & Stavi (1986) the fact that they get the truth conditions right, and one of our complaints against Geurts & Nouwen (2007) was the fact that they don't. But if, as we claim, the meaning of (1-a) is not a proposition, how can it get *any* truth conditions, let alone the correct ones?

We have argued that (1-a) means that the minimal n s.t. the speaker GRANTs that John petted exactly n rabbits is three. As we have seen in (59), from this it follows that the speaker makes the following three assertions:

- (60) a. John did not pet exactly two rabbits.
 b. John did not pet exactly one rabbit.
 c. John did not pet exactly zero rabbits.

This interpretation of (1-a) accounts straightforwardly for cases when it is false. Suppose, for example, that John petted exactly two rabbits. Then the content of the assertion (60-a) is false, which accounts for the falsity of (1-a).

Things are more interesting when the sentence is true. Suppose John petted exactly four rabbits—then (1-a) ought to be true. Indeed, the content of all the assertions in (60) would be true. Is this sufficient to account for the truth of (1-a)?

Not quite. If the minimal n s.t. the speaker GRANTs that John petted exactly n rabbits is three, it follows that the speaker makes the assertions in (60), but it does not follow that these are *all* the assertions that she makes. The speaker could, for example, also assert (61).

(61) John did not pet exactly four rabbits.

In this case, it would still be the case that the minimal n s.t. the speaker GRANTs that John petted exactly n rabbits is three, yet (1-a) would be false, rather than true.

In order to account for the truth of (1-a), then, we need to rule out assertions like (61). We cannot do so on logical grounds, since an assertion of (61) would be perfectly consistent with the interpretation of (1-a) we are proposing. However, we *can* rule it out on pragmatic grounds, specifically by conversational implicature.

By using a superlative quantifier, the speaker took the trouble to indicate that she accepts the commitments of all the assertions in (60); if she also wanted to commit to the claim that John did not pet exactly n rabbits for other values of n , the maxim of Quantity dictates that she should have indicated that as well. From the fact that she didn't, we can conclude, by a straightforward implicature, that she is not committed to such an assertion. Since the content of all the assertions that the speaker *is* committed to is true, it follows that (1-a) is true if John petted exactly four rabbits, which is the result we want.

Note that, according to our view, what (1-a) says about values of $n < 3$ is an entailment, while what it says about $n \geq 3$ is an implicature.

This asymmetry between the falsity of the sentence (which follows semantically) and its truth (which follows pragmatically) captures the intuition (which also underlies Geurts & Nouwen (2007)) that when

one says (1-a) one doesn't know what the number of rabbits that John petted is; but one does know what that number is *not*.

The proposal that the truth of sentences with superlative quantifiers comes from implicature has received some experimental support. Hacoen *et al.* (2011b; 2011a) have presented subjects with pictures and sentences with superlative quantifiers, and asked subjects to indicate whether the sentences accurately describe the pictures. Reaction times for (correct) true judgments turned out to be significantly longer than reaction time for (correct) false judgments. In contrast, no significant difference was found between true and false judgments with comparative quantifiers. On the assumption that the computation of implicature takes additional time Bott & Noveck (2004), our theory provides a natural explanation of these results, which would be mysterious under Geurts & Nouwen's account.

The fact that the truth of superlative quantifiers follows pragmatically can also explain another difference between them and comparative quantifiers. Suppose John petted over a thousand rabbits. Then an utterance of (1-b), reproduced below, would clearly be rather odd.

(62) John petted more than two rabbits.

However, we believe it is unquestionable that (62) is true in the situation described. It is usually odd to talk about more than n individuals satisfying a certain property, when the number of individuals actually satisfying the property is substantially higher than n . However, in the right context, such a statement may be acceptable. Here are a few examples, taken from the Web:

- (63) a. If one person throws one piece of rubbish on the ground per day then that person throws 365 pieces of rubbish in one year, but Koh Tao has more than one person, in fact 320,000 people.⁸
 b. In some instances, your e-mail message may be seen by more than one person. . . in fact, it may be forwarded to the entire mailing list.⁹
 c. I was experiencing this strange feeling with more than one person; in fact, with lots of people of both sexes.¹⁰

Note that all these sentences would become odd if we replace the com-

parative *more than one person with at least two people*. This corresponds to the fact that it is much harder to judge a sentence with a superlative quantifier as true in such a situation.

This is easily explained if the truth comes from an implicature. A speaker uttering (1-a) is taken to assert that John did not pet exactly 2, 1, or 0 rabbits. As the speaker leaves it open whether John did not pet exactly 3, 4, 5. . . rabbits, the implicature arises that the speaker considers it possible that John did pet exactly 3, 4, 5. . . rabbits. But this implicature may get weaker the higher the numbers get, because there might be additional reasons why the speaker might consider it impossible that John petted, say, 1000 rabbits—and then, in fact, would be ready to assert that John did not pet 1000 rabbits.

The same sort of asymmetry can be seen, perhaps even more clearly, when we consider other speech acts besides assertion. Take requests, for example:

(64) Give me at least three cookies.

The conditions under which this request is *not* satisfied are clear: if the hearer gives the speaker fewer than three cookies—two, one, or no cookies at all. But the conditions under which the request *is* satisfied are less clear: not any number $n \geq 2$ of cookies will equally satisfy the speaker. Four cookies may be better than three, but one thousand may be too many.

3.3. Non-numerical scales

Superlative quantifiers do not always apply to numerical scales. Consider the following attested examples, where the *if not* continuation makes it clear what the relevant scale is:

- (65) a. Rehabilitation without invasive procedures is at least comparable if not superior to more invasive and costly procedures.¹¹
 b. This is at least misleading, if not wrong.¹²
 c. The agent who bills such expenses is at least unethical, if not criminal.¹³
 d. [That] prices continued to rise after the announcement. . . is at least possible if not certain in at least some parts of

- Russia.¹⁴
- e. The “Gab Drag” is at least plausible, if not necessary, from a dramatic perspective.¹⁵
- f. This is at least confusing, if not conflicting.¹⁶

This interpretation is attested even when a numeral is involved. As Horn (1972) and Kadmon (1987) have shown, (1-a) and (2-a) have readings that can respectively be roughly paraphrased as:

- (66) a. John petted exactly three rabbits, and maybe he petted more animals.
 b. Maybe John petted exactly three rabbits, but he petted no other animal.

In these examples, the superlative quantifier applies to entailment (Horn) scales: being superior entails being comparable, being wrong entails being misleading, being criminal entails being unethical, etc.

We formalize this phenomenon by generalizing ‘ \geq ’ beyond numerical comparisons: if P and Q are propositions, and P is higher on the relevant Horn scale than Q , we write $P \geq Q$.

This phenomenon is a generalization of our previous treatment of superlative quantifiers over numerals. To see this, note that one of the equivalent formulations of (1-a) is (67).

$$(67) \quad \forall n(\text{GRANT}(|\text{rabbit} \cap \lambda x.\text{pet}(j, x)| = n) \rightarrow n \geq 3)$$

Now, any proposition of the form $|\text{rabbit} \cap \lambda x.\text{pet}(j, x)| = n$, for $n \geq 3$, entails $|\text{rabbit} \cap \lambda x.\text{pet}(j, x)| \geq 3$. Hence, we can rewrite (67) as (68), where it is assumed that context restricts P to range only over statements about the number of rabbits that John petted.¹⁷

$$(68) \quad \forall P(\text{GRANT}(P) \rightarrow P \geq \wedge |\text{rabbit} \cap \lambda x.\text{pet}(j, x)| \geq 3)$$

Similarly, (2-a) can be formulated as (69-a), which is equivalent to (69-b).

- (69) a. $\forall n(\text{GRANT}(|\text{rabbit} \cap \lambda x.\text{pet}(j, x)| = n) \rightarrow n \leq 3)$
 b. $\forall P(\text{GRANT}(P) \rightarrow P \geq \wedge |\text{rabbit} \cap \lambda x.\text{pet}(j, x)| \leq 3)$

However, we will continue to use the conceptually simple numerical comparison, whenever the scale is numerical.

4. ACCOUNTING FOR THE DATA

Our proposed account can explain the facts motivating Geurts & Nouwen’s approach, and also the facts that constitute challenges to their theory.

For the purposes of this paper, we accept Geurts & Nouwen’s account of comparative quantifiers: in particular, that they are restricted to combine with first-order properties only. We therefore accept their account of the distributive facts concerning such quantifiers.

Regarding superlative quantifiers, we have seen that our theory provides truth conditions that are extensional and intuitively correct, something that Geurts & Nouwen fail to do. We have also seen that our approach, unlike theirs, does justice to the superlative morphology of superlative quantifiers, by paraphrasing (1-a) and (2-a) as (70-a) and (70-b) respectively.

- (70) a. The minimal (“least large”) number n s.t. the speaker GRANTS that John petted exactly n rabbits is 3.
 b. The maximal (“most large”) number n s.t. the speaker GRANTS that John petted exactly n rabbits is 3.

Recall that Geurts & Nouwen point out that (1-a) and (2-a) would be odd if we knew that John petted exactly three rabbits, but for different reasons: the former is false, whereas the latter is literally true but odd. Under the proposed approach, the reason for both is the same: if the speaker knows that John petted exactly three rabbits, then she ought to assert this, and not GRANT that John petted any other number of rabbits. However, both (1-a) and (2-a) implicate that the speaker GRANTS, but does not assert, that John petted exactly three rabbits, and that the speaker GRANTS this statement for other values of n ($n > 3$ for (1-a), $n < 3$ for (2-a)). Since this is an implicature, and not an entailment, both (1-a) and (2-a) are infelicitous rather than false, and the experimental results reported by Geurts et al. (2010) are explained.¹⁸

We can also explain why *at most*, but not *at least*, licenses NPIs. Fol-

lowing Kadmon & Landman (1993) and Krifka (1991; 1995), an NPI introduces alternatives, and implicates that all the stronger alternatives than the one asserted are false. For example, *anything* introduces alternative properties, but denotes the most general property: **thing**. Usually, all the other properties are stronger, hence are implicated to be false. But this is a contradiction: how can something be a **thing**, without having any more specific property? Therefore, the use of *any* is unacceptable in most environments. However, in a downward entailing context, the other alternatives are weaker: for example, under negation, not being a **thing** is stronger than not having any more specific property. Therefore, the alternatives are not implicated to be false, and *any* is licensed.

Now consider (14), repeated below:

- (71) a. At least three people have ever been in this cave (last century).
 b. At most three people have ever been in this cave (last century).

The NPI *ever* introduces alternative sets of times within the reference time—last century—and denotes the most general of them—the entire century. The clause inside the scope of the superlative quantifier implicates that all the stronger alternatives are false.

In order to explain why the NPI is licensed in (71-b), we need to show that (72-a) is stronger than (72-b)

- (72) a. At most 3 people have been in this cave at some time or other.
 b. At most 3 people have been in this cave last year.

Since we propose that superlative quantifiers are illocutionary operators, we need to define the notion of relative strength for speech acts. A natural definition is the following:

- (73) A_1 is as strong as or stronger than A_2 iff for all commitment spaces C , $\mathbb{U}(C + A_1) \subseteq \mathbb{U}(C + A_2)$

In words, A_1 is stronger than A_2 iff every commitment that A_1 creates is also a commitment that A_2 creates, but not vice versa.

In order to show that (72-a) is stronger than (72-b), one more piece of the puzzle is needed. A speaker who asserts ϕ is committed to the truth of ϕ ; but what if ϕ entails ψ ? Is the speaker also committed to the truth of ψ ? It is arguable whether, in general, this is a reasonable requirement, since often it is very hard to figure out the entailments of a proposition. However, we certainly *can* require this if the entailment from ϕ to ψ is, in some sense that we will not try to make precise, clear and obvious. In this case, indeed, the consistency requirement on commitment spaces requires that a speaker who asserts ϕ is committed to the truth of ψ .

The update with (72-a) is (74-a), and the update with (72-b) is (74-b).

- (74) a. $C + \bigwedge_{n>3} \text{ASSERT}(\neg \text{'n people have been in this cave at some time or other'})$
 b. $C + \bigwedge_{n>3} \text{ASSERT}(\neg \text{'n people have been in this cave last year'})$

Now, for any n , it is clear and obvious that $\neg \text{'n people have been in this cave at some time or other'}$ entails $\neg \text{'n people have been in this cave last year'}$; therefore, if a commitment state contains the commitments of the former, it ought, by consistency, also to contain the commitments of the latter. We can therefore conclude that the commitment space in (74-a) is a subset of the commitment space in (74-b), i.e. the former is stronger than the latter, which explains the licensing of NPIs with *at most*.

In order to explain why (71-a) is bad, we need only point out that (75-a) is not stronger than (75-b).

- (75) a. At least 3 people have been in this cave at some time or other.
 b. At least 3 people have been in this cave last year.

Intuitively, if the speaker minimally GRANTS that 3 people have been in this cave last year, then by a simple logical inference, the speaker must be prepared to minimally GRANT that 3 people have been in this cave at some time or other.

In order to account for the missing readings of superlative quanti-

fiers, as exemplified by (7-b), we need to discuss more thoroughly the phenomenon of embedded superlative quantifiers.

5. EMBEDDING

5.1. Constraints on embedding

Because Keenan & Stavi (1986) claim that superlative quantifiers are just generalized quantifiers, their theory predicts that they should be freely embeddable. This prediction is not borne out, as pointed out by Geurts & Nouwen. They present the following examples:

- (76) a. $\left. \begin{array}{l} \text{Each} \\ \text{Most} \\ \text{?About five} \\ \text{*None} \end{array} \right\}$ of the guests danced with at least/most
three of the waitresses.
- b. ?Betty didn't have at least/most three martinis.

When superlative quantifiers are replaced in these examples by epistemic modals, they claim, the judgments are the same. Geurts & Nouwen do not explain why epistemic modals behave in this way, but consider this fact evidence for their theory, according to which superlative quantifiers *are* epistemic modals.

We would be hesitant to draw such a conclusion based on only two examples. But even if it is true that superlative quantifiers are embedded in the same environments in which epistemic modals are (which is an empirical question), all that such a fact would demonstrate is that there is something similar in the meanings of superlative quantifiers and epistemic modals. In fact, it is often claimed that at least some epistemic modals are speech act modifiers themselves (see Cohen (2010) for a recent argument to that effect). If, as we believe, superlative quantifiers are also speech act modifiers, this would explain the similarity between them.

What is the prediction of our theory regarding embeddings of superlative quantifiers? At first sight, it might appear that we predict that embedding is impossible, since superlative quantifiers are speech act modifiers. But this is not quite right: as we have seen in section 2 (see also Krifka (to appear)), although there are contexts where speech acts

operators cannot be embedded, there are also contexts where they *can*.

We therefore ought to investigate the contexts under which superlative quantifiers can be embedded, and those where they can't, and see whether this distribution can be accounted for under the assumption that they are illocutionary operators. Let us look at a few examples.

5.2. Downward entailing environments

5.2.1. Quantifiers

On the basis of (76-a), Geurts & Nouwen claim that superlative quantifiers are good in the scope of strong quantifiers, but bad in the scope of weak ones. But the use of the partitive *Q of the guests* actually prefers the strong reading of the quantifiers, so this conclusion is dubious.

Note that the only quantifier that is really bad in (76-a) is *none*, whose scope is a downward entailing environment; in order to explain the unacceptability of the sentence, we need to look more closely at the behavior of superlative quantifiers in such environments.

5.2.2. Superlative quantifiers are often bad...

As example (76-b) indicates, superlative quantifiers are not good in the scope of negation. Nilsen (2007) observes that superlative quantifiers are generally bad in downward entailing contexts:

- (77) a. ??John hardly ate at least three apples.
b. ??Policemen rarely carry at least two guns.
c. ??This won't take at least 50 minutes.

Compare this with the acceptability of comparative quantifiers in the same environments:

- (78) a. John hardly ate more than three apples.
b. Policemen rarely carry more than two guns.
c. This won't take more than 45 minutes.

As we have seen, Geurts & Nouwen's explanation, namely that superlative quantifiers behave like epistemic modals, is not satisfactory.

Alternative approaches are taken by Büring (2007) and Cummins & Katsos (2010), who can be seen as proposing more sophisticated ver-

sions of Keenan & Stavi's (1986) ideas. While there are important differences between the two theories, both propose, in essence, that (1-a) is interpreted as (79).

(79) John petted three or more rabbits.

Büring and Cummins & Katsos demonstrate how the use of the disjunction can explain many of the puzzling facts about superlative quantifiers discussed above.

However, these theories, just like Keenan & Stavi (1986), predict that superlative quantifiers can be freely embeddable; in particular, they predict that superlative quantifiers can occur in the scope of downward entailing contexts. Indeed, although *at least three* is bad in downward entailing contexts, *three or more* is perfectly good, as can be seen by the following attested examples:

- (80) a. [A]gainst Pittsburgh, five players had two receptions, and nobody had three or more.¹⁹
 b. Nobody wants three or more inboxes and calendars in the government.²⁰
 c. Nobody wants to spend three days or more in hospital if they could be safely back home within 24 hours.²¹
 d. Five of you managed to guess two numbers correctly, but alas nobody got three or more right.²²

None of the previous theories, then, successfully accounts for the inability of superlative quantifiers to be under the scope of downward entailing operators. But what, then, is the explanation of this fact?

To answer this question, let us consider the meaning of the affirmative counterpart to (76-b):

(81) Betty had at least three martinis.

According to the theory proposed here, (81) means that the speaker asserts, for all values of $n < 3$, that it is false that Betty had exactly n martinis.

Therefore, if Betty had fewer than three martinis, (81) is false. If Betty had n martinis, for some $n \geq 3$, the sentence is true, but this truth follows pragmatically, by way of conversational implicature,

rather than semantically. The speaker could have also denied that Betty had n martinis, and this would have been a stronger statement than the one she actually chose to make. But the fact that the speaker chose not to make that statement, implicates that the speaker actually GRANTS, for all $n \geq 3$ ²³, that Betty had n martinis. Since we assume that the speaker is committed to the disjunction of all her contextually relevant GRANTS, it follows that she is committed to the disjunction, one of whose elements is n , and (81) is therefore true.

Thus, the falsity of (81) follows semantically, but its truth requires a scalar implicature. So (81) does not have standard truth conditions: to get a proposition from it, we need a scalar implicature. The implicature gets us a strengthened reading, which does have truth conditions.

It is well established that scalar implicatures do not survive when triggered in downward entailing contexts. For example, implicature usually causes disjunction to receive an exclusive interpretation. But in the antecedent of a conditional, disjunction is usually interpreted inclusively, because the implicature doesn't apply:

(82) If you drink or smoke, you will become ill

Sentence (82) cannot mean that one who drinks *and* smokes will escape illness!

In fact, Chierchia (2004) observes that scalar implicature cannot be embedded in any context that licenses *any*, and explains this fact as follows. Normally, the reading derived by scalar implicature is stronger than the literal meaning. For example, exclusive disjunction is stronger than inclusive disjunction. However, in contexts that license *any*, the statement derived by the implicature is actually weaker than the original statement, and this is not allowed, by what he calls "the Strength Condition". Thus, (83), in which disjunction is exclusive, is actually weaker than (82), and this is why this interpretation is not generated.

(83) If you drink or smoke but not both, you will become ill.

What about superlative quantifiers? Since their complete truth conditions are generated by scalar implicature, it follows that it should not be possible to embed them in a downward entailing context.

Let us, for concreteness, see how this comes about in the case of negation. The formalization of the affirmative (81) is:

- (84) $C + \sim \text{GRANT}(|\text{martinis} \cap \lambda x.\text{have}(\mathbf{b}, x)| = 2)) \wedge$
 $\sim \text{GRANT}(|\text{martinis} \cap \lambda x.\text{have}(\mathbf{b}, x)| = 1)) \wedge$
 $\sim \text{GRANT}(|\text{martinis} \cap \lambda x.\text{have}(\mathbf{b}, x)| = 0))$

If we treat (76-b) as the denegation of (81), and apply de Morgan and the complement rule, its formalization would be:

- (85) $C + \text{GRANT}(|\text{martinis} \cap \lambda x.\text{have}(\mathbf{b}, x)| = 2)) \vee$
 $\text{GRANT}(|\text{martinis} \cap \lambda x.\text{have}(\mathbf{b}, x)| = 1)) \vee$
 $\text{GRANT}(|\text{martinis} \cap \lambda x.\text{have}(\mathbf{b}, x)| = 0))$

What are the derived truth conditions? As before, they ought to be determined by what the speaker GRANTS. But we have, in fact, very little information about this. We know that the speaker GRANTS at least one statement with $n \leq 2$, but we don't know which. And the speaker may GRANT other statements, with various values of n —this is not precluded by (85). Thus, we can say very little about the truth conditions of (76-b).

Suppose Betty had exactly two martinis; if the speaker GRANTS the corresponding statement, her utterance would be true; but we don't know this. Now suppose Betty had exactly four martinis. Then we know that one of the statements the speaker GRANTS is false; but perhaps she also GRANTS that Betty had exactly four martinis, which would be true. In short, unlike the case of non-negated superlative quantifiers, truth conditions are not defined for negated ones, either semantically or pragmatically. This is why negated superlative quantifiers are bad.

We can now explain why (7-b), repeated below, does not have the reading where *at most* is under the scope of *may*.

- (86) You may have at most two martinis.

The scope of *may* licenses *any*, as demonstrated by (87).

- (87) You may pick any card.

This is explained by the fact the purpose of permissions is to lift restrictions for the addressee: a permission to pick a card with unspecified characteristics is stronger than a permission to pick, say, the Ace of Spades, since it gives the addressee more options.

Chierchia (2004) points out that, indeed, implicatures do not survive under *may*: (88) does not mean that you are not allowed to both drink and smoke.

- (88) You may drink or smoke.

Since implicature is crucial to the derivation of the truth conditions of sentences involving superlative quantifiers, the reading of (86) where it is under the scope of *may* is uninterpretable, and this is why it is not attested. If *at most* takes scope over *may*, there is, of course, no problem.

If superlative quantifiers are bad in downward entailing environments, it follows that they are positive polarity items. Indeed, they behave similarly to PPIs. For example, it is well known that PPIs are typically not good with positive questions, but require negative questions (Borkin 1971; Pope 1972):

- (89) a. $\left\{ \begin{array}{l} *Is \\ Isn't \end{array} \right\}$ it rather cold for this time of year?
 b. $\left\{ \begin{array}{l} *Are \\ Aren't \end{array} \right\}$ you pretty tired?
 c. $\left\{ \begin{array}{l} *Does \\ Doesn't \end{array} \right\}$ he have TONS of money?²⁴

The same behavior is observed when a superlative quantifier is embedded under a question operator:

- (90) $\left\{ \begin{array}{l} *Did \\ Didn't \end{array} \right\}$ John have at least three martinis?

Of course, in an echoing or contrast context, a superlative quantifier can be acceptable in a positive question. Suppose that at a meeting of Alcoholics Anonymous, it was decided that anyone who drank no more than three martinis would be honored by a public mention of the person's name. We know John to be a very heavy drinker, so when we hear his name announced, we can certainly ask, incredulously:

- (91) Did John have at most three martinis?

5.2.3. ...but sometimes good

The facts concerning embeddings of superlative quantifiers in downward entailing environments are not so simple, however. There are cases when superlative quantifiers are actually perfectly good in such environments, as exemplified by the following minimal pair, adapted from Nilsen (2007):

- (92) a. Click away at the *Finalize your order* button: You will get a discount if you click it at least twice.
 b. ?? Don't click incessantly on the *Finalize your order* button: You will generate multiple orders if you click it at least twice.

The difference in acceptability between (92-a) and (92-b) is striking: yet they appear to make very similar statements, and in both, the superlative quantifier is in a downward entailing environment. The only difference appears to be that in (92-a), the consequences of clicking multiple times on the button are “good”, whereas in (92-b), the consequences are “bad”.

What counts as “good” or “bad” consequences is, of course, not easy to define. Yet, it is generally fairly clear intuitively what the speaker judges to be good or bad. We are not claiming that it is a strict rule that superlative quantifiers *never* occur in the antecedents of conditionals with “bad” consequents, but the tendency is quite strong. Indeed, a corpus study Shapira (2010) reveals that only 5.7% of conditionals with *at least* in the antecedent have “bad” consequents. Shapira found that, in contrast, 45.05% of conditionals with *more than* in the antecedent have “bad” consequents.

Nilsen (2007) attempts to account for this phenomenon by stipulating that *at least n* presupposes that *n* is the least useful of the alternatives. But apart from being stipulative, this is incorrect: in (1-a), there is no reasonable sense in which three rabbits is the least “useful” among the alternatives.

How, then, can we account for these facts?

To answer this question, let us consider the antecedent of the conditional as an asserted full sentence:

- (93) You will click at least twice.

According to our theory, (93) means that the speaker asserts that the hearer will not click exactly *n* times for $n = 0$ and $n = 1$.

As we have seen, if the hearer clicks exactly zero or one times, (93) is false. If the hearer clicks $n \geq 2$ times, (93) is true, but its truth follows pragmatically, by way of conversational implicature, rather than semantically: by not explicitly excluding higher values of *n*, the speaker implicates that she is only committed to the assertions that $n \neq 0$ and $n \neq 1$; since the contents of both these assertions are satisfied for $n \geq 2$, (93) is true.

Thus the falsity of (93) follows semantically, but its truth requires an implicature. So (93) does not have standard truth conditions: to get a proposition from it, we need a scalar implicature. Since scalar implicature cannot be embedded in downward entailing contexts, superlative quantifiers are predicted to be bad in such environments.

Yet, as we have seen, this prediction is not borne out. While (92-b) is indeed bad, (92-a) is fine. How can we explain this fact?

If there were an alternative way to interpret (93), one that would provide us with truth conditions without implicature, it would be possible to embed the superlative quantifier in a downward entailing environment. In other words, we need to get the strengthened meaning of (93), namely that you click twice or more, without implicature.

It so happens that *at least* does have such an alternative interpretation. Kay (1992) identifies three senses of *at least*, one of which is particularly relevant here.²⁵

This is the interpretation Kay calls the *evaluative* sense of *at least*, which can be exemplified by the following sentence:

- (94) At least this hotel is centrally located.

Several points should be made regarding this sense.

First, note that (94) presupposes that being centrally located is a desirable property for hotels. If this presupposition is not satisfied, as in (95), the sentence is odd.

- (95) # At least this hotel is noisy.

Kay describes the evaluative use of *at least* when applied to a root clause. When *at least* is embedded in the antecedent of a conditional, the felicity of the evaluative reading requires that the property be a

“positive” one; but sometimes we can only tell this from the consequent. For example, there is nothing inherently good or bad about being on Walnut St.; the felicity of the following sentences depends on the “goodness” of the consequent: taking the hotel is “good”, going elsewhere is “bad”.

- (96) a. If this hotel is at least on Walnut St., I will take it.
 b. #If this hotel is at least on Walnut St., I will go elsewhere.

Second, it follows from (94) that the hotel is centrally located, and that being centrally located is the minimal requirement for the speaker’s goals (presumably, staying at the hotel), though the hotel may possess additional good qualities.

Third, Kay points out that the property of being centrally located is not the maximally positive one, and there are some positive properties that the hotel fails to have. In our view this last point is implicated, rather than entailed by (94), because it can be canceled:

- (97) You should consider this hotel. At least it’s centrally located, and possibly it’s the perfect hotel for you.

Compare this with the impossibility of canceling the entailment that being centrally located is the minimal requirement:

- (98) #This hotel is at least centrally located, but it would be ok even if it were far away.

Fourth, the evaluative reading, unlike the reading of *at least* we discussed up to now, ought not to be analyzed as a speech act modifier. One piece of evidence for this is the fact that (94) is perfectly felicitous even when the speaker knows all the properties of this hotel, whereas, as pointed out above, (1-a) would be odd if the speaker knew exactly how many rabbits John petted.

Kay discusses briefly the syntax of the evaluative *at least*, and argues that it functions as an unfocused parenthetical: “Initial, final, and preverbal position are favorite places for parenthetical insertions in English, though they are not the only such positions available” (p. 318). We assume that, perhaps depending on intonation and context, the evaluative reading of *at least* is, in principle, always available.

We suggest that embedding a sentence such (93) in the scope of a conditional is another way to force it to receive an evaluative interpretation, since the illocutionary reading is not available. The resulting meaning is as follows:

1. Presupposition: Clicking (exactly) twice is good
2. Implicature: You will click less than some maximally good limit
3. Entailment: Two is the minimal number of clicks you will perform.

When (93) is embedded in a downward entailing environment, as in (92-a), the presupposition is satisfied, since getting a discount is a good thing. As discussed above, the implicature disappears, because of the Strength Condition. We are left with the entailment, which gives us the desired truth conditions: if twice is the minimal number of clicks you will perform, you will get a discount.

In contrast, we cannot embed (93) in (92-b), because, in this case, the presupposition is not satisfied: clicking twice is not good, because it will generate multiple orders.

There is further evidence that the superlative quantifiers in these examples really get the evaluative reading and not the illocutionary interpretation such quantifiers usually receive. Recall that one of the characteristics of superlative quantifiers, as opposed to comparative ones, is that they are not ambiguous under a deontic modal. So, (99) can only mean that the hearer is not allowed to have more than one drink, not that it’s ok if the hearer has no more than one.

- (99) You may have at most one drink

However, this reading *is* available with superlative quantifiers in the antecedents of conditionals with “good” consequents. For example, suppose Mary is invited to a party. She is a very moderate drinker, so she decides that if she is allowed to have no more than one drink, she will come. Mary can then say:

- (100) If I may have at most one drink, I will come to the party.

Note that, in this case, the superlative quantifier is in the scope of *may*, which it is not supposed to be able to do under the illocutionary inter-

pretation. This is an indication that the superlative quantifier receives a different reading. Since the consequent is “good”, this different reading is plausibly the evaluative interpretation.

In contrast, note what happens if the consequent is “bad.” Suppose John is also invited to the party. Unlike Mary, John believes that a party is not good unless there is plenty of alcohol, and demands that guests not be allowed to have fewer than two drinks. Yet, he cannot say:

(101) #If guests may have at most one drink, I will not come to the party.

The reason is that the consequent is “bad”, so the superlative quantifier cannot be in the antecedent of the quantifier,

Kay (1992) does not discuss the possibility of an evaluative reading for the other superlative quantifier, *at most*, but it seems to have an evaluative interpretation too:

(102) This is a bad hotel; at most, it’s centrally located.

Just as with *at least*, being centrally located is still presupposed to be a good thing. The difference is in the entailment: being centrally located is less than the minimal requirement, so that the sentence entails that nothing better than being centrally located can be said about the hotel.

Now consider (103).

(103) You will click at most twice

If (103) receives the evaluative reading, the result is this:

1. Presupposition: Clicking (exactly) twice is good
2. Implicature: you will click more than some maximally bad limit
3. Entailment: the maximal number of clicks you will perform is two.

When (103) is embedded as in (104-a), the presupposition is satisfied, the implicature disappears, and the entailment provides us with the correct truth conditions: if two is the maximal number of clicks you perform, you will get a discount. This is impossible in (104-b), because the presupposition is not satisfied.

(104) a. If you click at most twice, you will get a discount.
b. # If you click at most twice, the transaction will be canceled.

The behavior of both superlative quantifiers in the antecedent of a conditional is thereby explained.

Interestingly, strong NPIs behave in a way that is exactly the opposite of that of superlative quantifiers—they are fine if the consequent is “bad”, but ruled out if the consequent is “good” Lakoff (1969):²⁶

(105) a. If you eat $\left\{ \begin{array}{l} \text{some} \\ *any \end{array} \right\}$ spinach I will give you \$10
b. If you eat $\left\{ \begin{array}{l} *some \\ any \end{array} \right\}$ candy I will whip you.

Regine Eckardt (pc) shows that the same is true of strong NPIs:

(106) If you budge an inch, I will $\left\{ \begin{array}{l} \text{kill} \\ *thank \end{array} \right\}$ you.

We have already seen that superlative quantifiers are PPIs; it is tempting to speculate that our account can be extended to other PPIs, i.e. that they have an evaluative interpretation too. Possibly, NPIs also have something like an evaluative interpretation, except that they presuppose that an undesirable, rather than a desirable property. However, we will not explore these intriguing possibilities further here.

Nilsen (2007) notes that superlative quantifiers in the restrictor of a universal behave the same way as in the antecedent of a conditional—they are acceptable if the consequent is “good”, but are ruled out if it is “bad”:

(107) a. Everybody who donates $\left\{ \begin{array}{l} \text{more than} \\ \text{at least} \end{array} \right\}$ 10 BGN will get a thank you postcard.
b. Everybody who uses $\left\{ \begin{array}{l} \text{more than} \\ \# \text{at least} \end{array} \right\}$ three exclamation marks is a fool.

The explanation we propose is the same as we have proposed for conditionals. The restrictor of a universal is a downward entailing con-

text, in which conversational implicatures are canceled: the quantification domain of (108) includes individuals who drink *and* smoke.

(108) Everybody who drinks or smokes will become ill.

Therefore, only the evaluative sense of superlative quantifiers is allowed, and it requires that the nuclear scope will be perceived to be positive. Since getting a thank you postcard is a good thing, (107-a) is fine; but since being a fool is bad, (107-b) is odd.

If the consequent of the conditional is a deontic modal, there is a clear difference in acceptability, depending on the type of modal. Superlative quantifiers are fine if the consequent is a permission, but bad if the consequent is an obligation:

- (109) a. If you're $\left\{ \begin{array}{l} \text{more than} \\ \text{\#at least} \end{array} \right\}$ 30 minutes late you must report immediately to the Office of the Registrar, Room 2122, South Building Nilsen (2007).
 b. If you arrive at least 30 minutes early, you may come to my office for a coffee.
- (110) a. If you have $\left\{ \begin{array}{l} \text{less than} \\ \text{\#at most} \end{array} \right\}$ \$50 in your pocket, you ought to go to the bank to get more.
 b. If you make at most \$500 a month, you may apply for a stipend.

Again, the explanation appears to be related to the good/bad distinction. In general, obligations are “bad,” but permissions are “good.” Thus, the presupposition of the evaluative sense of superlative quantifiers is satisfied with obligations, but violated with permissions.

The same phenomenon obtains regarding superlative quantifiers in the restrictor of quantifiers: obligations are considered “bad,” whereas permissions are “good”:

- (111) a. Campaign finance laws make it a campaign's responsibility to disclose the occupation and employer of everybody who contributes $\left\{ \begin{array}{l} \text{more than} \\ \text{\#at least} \end{array} \right\}$ \$200 (Nilsen 2007)

- b. Everybody who contributed at least \$200 may vote in the Primaries.
 c. Every man who had $\left\{ \begin{array}{l} \text{more than} \\ \text{\#at least} \end{array} \right\}$ two martinis must refrain from driving.

Epistemic modals present an interesting problem. Geurts & Nouwen note that superlative quantifiers may be embedded in the antecedent of a conditional, if the consequent contains an epistemic modal, either of necessity or possibility:

- (112) If Betty had at least three martinis, she must/may have been drunk.

They admit, however, that their theory cannot handle this example.

Similar facts obtain for superlative quantifiers in the restrictor of a quantifier:

- (113) Everybody who had at least three martinis, must/may have been drunk.

As we have seen, the antecedent of a conditional and the restrictor of a universal are downward entailing contexts, in which scalar implicature does not survive; since an implicature is necessary to derive truth conditions, the antecedent does not have derived truth conditions. One may wonder whether the superlative quantifier receives an evaluative interpretation, but this is implausible. Epistemic modals are neutral with respect to “good” or “bad”: there is nothing inherently “good” or “bad” about the speaker believing something to a high or low degree. Hence, the presupposition of the evaluative interpretation is not satisfied.

A possible solution to this behavior of superlative quantifiers may be provided by the type of conditionals that Sweetser (1996) calls *meta-metaphorical conditionals*. They are exemplified by the following:

- (114) a. If the Île de la cité is the heart of Paris, the Seine is the aorta.
 b. If life is a candle-flame, then people are the moths burned on the flame.

These conditionals relate two metaphors that are literally false (the *Île de la cité* is not a heart, the Seine is not an aorta, etc.). However, people may still refuse to assert their falsity, but accept them as metaphors, and be willing to assert them. The intended meaning appears to be that if the speaker is willing to assert the antecedent, she would also be willing to assert the consequent.

We propose that conditionals containing a superlative quantifier in the antecedent and an epistemic modal in the consequent are interpreted in a similar way: the conditional does not relate two propositions, but rather two speech acts.²⁷ It is the main claim of this paper that superlative quantifiers are interpreted as (meta) speech acts; hence the antecedent of (112), containing a superlative quantifier, is a speech act: the speaker denies that Betty had two martinis, denies that Betty had one martini, and denies that Betty had zero martinis. Following the theory of epistemic modals as illocutionary operators (Cohen (2010); cf. section 5.1 above), the consequent, too, is a speech act: it is an assertion that Betty was drunk, with a high (for *must*) or low (for *may*) degree of strength. What the conditional says is that if the speaker is willing to make the first speech act, then she will be willing to make the second speech act too.

In a similar way, if the speaker utters (113), she is saying that for every individual x s.t. she is willing to deny that x had n martinis if $n < 3$, she is willing to assert with a high/low degree of belief that x was drunk.

5.2.4. Back to negation

If the superlative quantifier in the antecedent of a conditional is negated, the judgments are reversed: if the consequent is “good” the sentence is bad, whereas if the consequent is “bad”, the sentence is good:

- (115) a. If you don’t click at least twice, the system won’t respond to your request.
 b. #If you don’t click at least twice, you will get a discount.

The reason is simple. Under the evaluative interpretation, clicking at least twice requires a “good” consequence; hence *not* clicking at least

twice requires a bad consequence.

Indeed, we receive the same behavior with unambiguously evaluative uses of *at least*:

- (116) a. If this hotel isn’t at least on Walnut St., I will go elsewhere.
 b. #If this hotel isn’t at least on Walnut St., I will take it.

In fact, this behavior obtains in general when the the superlative quantifier in the antecedent of a conditional is in the scope of a downward entailing operator, as can be seen by the following attested examples:

- (117) a. If nobody reveals a hand (that is, nobody has at least 3 of a kind) everyone still in may make another exchange.²⁸
 b. Ace-High, A five-card hand with an ace but no pair; if nobody has at least a pair, it’s the winning hand (similarly “King-high”, “Queen-high” etc.).²⁹

We have seen that since permissions are “good” and obligations are “bad”, superlative quantifiers are fine with the former but odd with the latter. When the superlative quantifiers are negated, these judgments are reversed: they are bad with permissions, but good with obligations.

- (118) a. If Betty didn’t have at least 3 martinis, she should be barred from our club.
 b. #If Betty didn’t have at least 3 martinis, she can drive.
 c. Everybody who didn’t drink at least 3 martinis should be barred from our club.
 d. #Everybody who didn’t drink at least 3 martinis can drive.
 e. If you don’t have at least \$50, you should go to the bank to get more.
 f. #If you don’t make at least \$500 a month, you may apply for a stipend.

The judgments are reversed with epistemic modals too, but for a different reason. We have seen that epistemic modals are fine with superlative quantifiers, because they naturally express a connection between two speech acts. However, if the superlative quantifier in the

antecedent is negated, the result is bad:

- (119) # If Betty didn't have at least three martinis, she must/may have been sober.

The reason is that (119), unlike (112) and (113), does not convey any natural connection between two speech acts.

Recall the discussion of (76-b), repeated below:

- (120) ?Betty didn't have at least three martinis.

As we have seen above, a negated superlative quantifier is a rather unclear statement. The antecedent of the conditional, (120), only says that the speaker GRANTS the statement that Betty had n martinis for some $n < 3$; but there may be other values of $n < 3$ for which she does *not* GRANT this, while there may be values of $n \geq 3$ for which she *does*. Hence, there is no clear connection between this statement and the speaker's level of confidence that Betty was sober.

As we have seen, (112) and (113) cannot be helped by the evaluative reading, since there is nothing inherently "good" or "bad" about epistemic modals. In fact, one may take this point even further. Why can't (120), on its own, receive the evaluative interpretation, and, consequently, be acceptable?

It turns out that, in general, the evaluative interpretation is bad in the scope of negation, regardless of whether the predicated property is "good" or "bad":

- (121) a. ??This hotel isn't at least centrally located.
b. ??This hotel isn't at least far away.

We are not sure what the reason for this behavior is.

5.3. Propositional attitudes

Propositional attitudes can, of course, allow for recursive embedding:

- (122) Annabelle believes that Matthew suspects that Darcy wants to kill Guy.

Can speech acts also be embedded under propositional attitudes?

They ought to be, if speech acts are sufficiently similar to propositional attitudes, i.e. if speaking is sufficiently similar to thinking (Krifka to appear). According to some, this is indeed the case: the only difference is that the speaker uses a natural language, whereas the thinker uses a *language of thought* (Fodor 1975).

We will not settle the language of thought issue here, nor do we need to: superlative quantifiers are meta-speech acts, which express willingness or unwillingness to make certain speech acts. Thus, meta-speech acts are clearly similar to propositional attitudes, and hence are predicted to be embeddable under them.

This prediction is, indeed, borne out:

- (123) Mary thinks that John petted at least three rabbits.

Note that (123) is, in fact, ambiguous, between two readings that can be paraphrased as (124-a) and (124-b).

- (124) a. Mary thinks: "John petted n rabbits," for some number n , and the speaker says that n is at least three.
b. Mary thinks: "John petted at least three rabbits."

The two readings are distinct: only (124-b) implicates that Mary is not sure how many rabbits John petted. How can this ambiguity be accounted for?

Note that (125), with a comparative quantifier, is similarly ambiguous.

- (125) Mary thinks that John petted more than two rabbits.

This ambiguity can straightforwardly be accounted for as involving the relative scopes of the attitude verb and the comparative quantifier.³⁰ This suggests that the ambiguity of (123) is also a scope ambiguity.

Specifically, in our system, (124-a) means that the minimal n s.t. the speaker GRANTS that Mary thinks that John petted exactly n rabbits is 3. Note that under this reading, the propositional attitude is inside the scope of the illocutionary operator. But (124-b), which is, in fact, much more prominent, means that the minimal n s.t. Mary GRANTS that John petted exactly n rabbits is 3. Now, the illocutionary operator

is inside the scope of the propositional attitude operator.

Recall that a speech act has (at least) two arguments, in addition to the propositional content: the speaker and addressee. We propose that the variable s , indicating the speaker, is a shiftable indexical in the sense of Schlenker (2002). According to Schlenker, propositional attitudes quantify not over possible worlds, but over contexts. Contexts determine a possible world, but also a speaker and addressee. Non-shiftable indexicals, such as the first person pronoun in English, can only refer to the speaker in the context of utterance. But shiftable indexicals, such as the first person pronoun in Amharic, may refer to the speaker of the embedded context. This is why (126) can only mean that John says that the speaker is a hero, while its counterpart in Amharic can mean that John says that John is a hero.

(126) John says that I am a hero.

Although first person pronouns do not shift in English, speaker-oriented expressions do seem to shift. For example, (127-a) means that the speaker considers the fact that it stopped raining lucky; but when embedded in (127-b), the sentence means that Mary considers it lucky.

- (127) a. Luckily it stopped raining.
b. Mary thought that luckily it had stopped raining.³¹

Treating the speaker variable of a speech act as a shiftable indexical, and ignoring the addressee, we can formulate³² the respective logical forms of readings (124-a) and (124-b) as follows (where c_0 is the context of utterance):

- (128) a. $c_0 : \forall n(\text{ASSERT}(\text{speaker}(c_0), \text{addressee}(c_0))$
 $(\text{think}_{c_1}(\mathbf{m}, (|\text{rabbits} \cap \lambda x.\text{pet}(\mathbf{j}, x)| = n))) \rightarrow n \geq 3)$
 b. $c_0 : \text{think}_{c_1}(\mathbf{m}, \forall n(\text{ASSERT}(\text{speaker}(c_1), \text{addressee}(c_1))$
 $(|\text{rabbits} \cap \lambda x.\text{pet}(\mathbf{j}, x)| = n) \rightarrow n \geq 3))$

Since c_1 is the context of Mary's thought, both "speaker" and "addressee" of c_1 are Mary herself, hence (128-b) means that Mary thinks to herself that John petted at least three rabbits. This is the intended interpretation.

5.4. Requirements

Sentence (129) is ambiguous between (130-a) and (130-b).

- (129) John needs at least three martinis.
 (130) a. The minimal n s.t. the speaker GRANTS that John needs exactly n martinis is $n = 3$.
 b. The minimal number n of martinis that satisfies John's needs is $n = 3$.

According to (130-a), John needs some number of martinis, and the speaker is not sure how many, but is sure it is no less than 3. Reading (130-b), which is probably more prominent, says that the minimal needs of John are satisfied with 3 martinis (though more might be better).

The corresponding formulations are as follows:

- (131) a. $\forall n(\text{GRANT}(\text{need}(\mathbf{j}, |M \cap \lambda x.\text{have}(\mathbf{j}, x)| = n)) \rightarrow n \geq 3)$
 b. $\text{need}(\mathbf{j}, \forall n(\text{GRANT}(|M \cap \lambda x.\text{have}(\mathbf{j}, x)| = n) \rightarrow n \geq 3))$

Geurts and Nouwen account for the first reading in a straightforward, compositional way. In order to get the second, more prominent reading, they propose a mechanism of modal concord: the epistemic modal of *at least* becomes deontic as a consequence of the explicit requirement indicated by *need*. In contrast, we account for the two readings of (129) as a straightforward case of scope ambiguity, without the need to posit any additional devices.

Note that in order to obtain the reading in (130-b), we are treating the verb *need* as a sort of meta-speech act. Specifically, the verb must be able to subcategorize for speech acts, i.e. the subject must be an entity that is capable of GRANTing (typically a human). Indeed, with an inanimate subject, this reading is not available:

- (132) The project needs at least three years to complete.

Sentence (132) can only get the reading corresponding to (130-a), namely that the speaker denies that the project will be completed in less than three years (but is not sure how long it will actually take). Crucially, it does not receive the reading corresponding to (130-b), where three years will definitely be enough to provide whatever is

necessary for the project to finish (though more time may perhaps be better).

Of course, the formulations in (131) are not complete, so long as we do not provide an analysis of the relation **need**. In fact, we argue that, together with such an appropriate definition, our approach provides the way to solve a difficult puzzle involving the meaning of expressions of requirements. Consider the following sentence:

(133) John needs to have a drink in order to go to sleep.

On the face of it, this sentence expresses a necessary and sufficient condition for John's going to sleep: if he has a drink he will fall asleep, and if he doesn't—he won't. But von Fintel and Iatridou (2005) point out that the condition, although necessary, is not sufficient: in order to go to sleep John also needs not to be working, to have a place to sleep on, to breathe, etc.

There are, however, two problems with the view that sentences like (129) express only necessary conditions. One problem is that every entailment of a necessary condition is also a necessary condition: for example, having a drink entails being alive. Now, although, upon reflection, most people will agree that John needs to be alive in order to sleep, (134) certainly does not sound very natural (cf. von Stechow et al. 2005).

(134) John needs to be alive in order to go to sleep.

Nouwen (2009) points out another problem, which is the following. If requirements are necessary conditions, (135-a) would entail (135-b), which would entail (135-c). From these, (136) would follow, but, again, this sounds wrong.

- (135) a. John needs three martinis to fall asleep.
 b. John needs two martinis to fall asleep.
 c. John needs one martini to fall asleep.

(136) The minimal number of martinis that John needs to fall asleep is 1.

If a sentence is true yet sounds funny, this is often indicative of an ambiguity: under one reading it is true, but under another, it is odd.

Perhaps this is the case with (134); this suggests an alternative way to interpret von Fintel and Iatridou's observation.

We maintain that requirements are, in fact, necessary and sufficient conditions. But we suggest that statements of requirements contain an implicit *at least*. Verbs like *need*³³ subcategorize for a nominal expression (DP) that expresses a minimal requirement that can be made explicit with *at least*.

Some evidence for this comes from examples such as (137-a), which can only mean that the number n such that you need at least n good deeds to go to heaven is $n \leq 3$. In contrast, (137-b) has a perverse reading saying that if you did more than three good deeds you won't go to Heaven.

- (137) a. You need at most three good deeds to go to Heaven.
 b. You need to have done at most three good deeds to go to Heaven.

Assuming an implicit *at least* in the representation of requirements, the result is ambiguous, depending on whether this superlative quantifier takes scope above or below the modal. Thus, (135-a) actually means (138), which is ambiguous between (139-a) and (139-b), with the latter being clearly the dominant reading.

(138) John needs at least three martinis (to fall sleep).

- (139) a. The set of necessary and sufficient conditions for John's falling asleep includes at least the condition of having three martinis (i.e. the conjunction of the necessary and sufficient conditions entails having three martinis).
 b. The necessary and sufficient condition for John's falling asleep is that he has at least three martinis

This ambiguity falls out of the formulations in (131), if the order ' \geq ' is generalized to entailment, as discussed in section 3.3 above. We formulate (139-a) and (139-b), respectively, as (140-a) and (140-b) (ignoring issues of modality):

- (140) a. $\forall P(\text{GRANT}(\text{sleep}(j) \leftrightarrow P) \rightarrow P \geq \hat{\ } | M \cap \lambda x.\text{have}(j, x) | \geq 3)$

- b. $\text{sleep}(j) \leftrightarrow \forall P(\text{GRANT}(P) \rightarrow P \geq \hat{M} \cap \lambda x.\text{have}(j, x) \geq 3)$

The formulation in (140-a) means that if the speaker GRANTS that P is a necessary and sufficient condition for John's sleeping, then P is greater than or equal to (i.e., entails) the condition that John have three martinis or more. In other words, the speaker knows that the set of necessary and sufficient conditions for John's falling asleep includes at least having three martinis.

The formulation in (12b) means that it is a necessary and sufficient condition for John's falling asleep that any statement that the speaker GRANTS (contextually restricted to statements about the number of martinis John has) is that John had a number of martinis that is greater than or equal to 3.

Note that (140-a) entails (134): if a necessary and sufficient condition for John's falling asleep entails having three martinis or more, and having three martinis or more entails breathing, then a necessary and sufficient condition for John's falling asleep entails breathing. However, (140-b) does not entail (134): if having at least three martinis is a necessary and sufficient condition for John's falling asleep, it does not follow that breathing is also a necessary and sufficient condition for John's falling asleep. We believe this is why most people would agree that (134) follows, but feel uncomfortable with this conclusion.

Similarly, (135-b) only follows from (140-a), but not (140-b). It follows from (140-a), for if the necessary and sufficient condition for John's falling asleep entails having three martinis or more, it entails having two martinis or more. But (135-b) does not follow under reading (140-b), for if having at least three martinis is necessary and sufficient for falling asleep, it does not follow that having at least two martinis is necessary and sufficient for John's falling asleep.

Therefore, (136) only follows under the following reading: the minimal number n s.t. the set of necessary and sufficient conditions for John's falling asleep entails having n martinis is 1. But since this reading is extremely awkward, most people would refuse to accept this inference.

6. CONCLUSIONS AND IMPLICATIONS

In this paper, we have argued that superlative quantifiers are quantifiers over meta-speech acts, and developed a framework for modeling speech acts and meta-speech acts. In this framework, GRANTing a proposition is the denegation of asserting its negation. This framework also provides a natural analysis of conjunction and, to some extent, disjunction of speech acts.

We have proposed that *at least 3 ϕ* means that the minimal number n s.t. the speaker GRANTS $\phi(n)$ is 3. And *at most 3 ϕ* means that the maximal number n s.t. the speaker GRANTS $\phi(n)$ is 3.

Furthermore, we argue that the falsity of a superlative quantifier is determined semantically, but its truth is determined pragmatically, via scalar implicature. Hence, superlative quantifiers can be embedded in environments where scalar implicature survives.

We have shown how this theory explains the facts concerning the distribution and interpretation of superlative quantifiers better than competing approaches, while maintaining correct, objective truth conditions.

If this account is on the right track, it has significant implications that go beyond an account of a particular linguistic phenomenon. We have argued that interlocutors often express meta-speech acts: they are not moves in the conversation, but indicate which moves are possible. Moreover, we have demonstrated that speech acts (including meta-speech acts) can be modeled as changes of commitment spaces. Thus they become semantic objects, and hence part of semantic recursion, although they do not have truth conditions. Therefore they can be embedded, provided their embedding is interpretable.

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Notes

¹There is another reading of (1-a) (Horn 1972; Kadmon 1987), which can be made the preferred interpretation by focus, where John petted exactly three rabbits, and maybe other animals as well. The two readings behave differently, e.g., with respect to anaphora; but we defer discussion of this reading until section 3.3.

²Geurts & Nouwen (2007) point out additional cases where the distribution of superlative quantifiers is more restricted than that of comparative quantifiers; we will discuss such cases when we deal with embedded superlative quantifiers in section 5 below.

³Incidentally, the meaning of (1-b) is different from that of (i), since only the latter implicates that John petted no more than two rabbits.

(i) John petted two rabbits.

⁴Note that this behavior obtains even when the superlative quantifier does not relate to a numeral:

(i) At $\left\{ \begin{array}{l} \textit{most} \\ \textit{*least} \end{array} \right\}$ some bats and small rodents have ever been in this cave

Non-numerical scales will be discussed in section 3.3.

⁵See Roelofsen & van Gool (2009) for more on the interpretations of disjunctive questions, and how they are affected by intonation.

⁶We thank Barbara Partee for this modification of our original example.

⁷The name we chose for this speech act is of course meant to be suggestive, but we are definitely not claiming that the English verb *grant* denotes the speech act GRANT. Similarly, the English verb *assert* does not denote the speech act ASSERT, but something much stronger.

⁸http://www.savekohtao.com/index.php?option=com_content&view=section&layout=blog&id=3&Itemid=4&lang=en

⁹<http://www.angelfire.com/mi/FAST/inputform.html>

¹⁰<http://www.circleofa.org/articles/FallingInLoveWithEveryone.php>

¹¹<http://blog.myphysicaltherapyspace.com/2010/08/is-acl-surgery-the-spinal-fusion-of-the-lower-extremity.html>

¹²<http://stackoverflow.com/questions/3815411/stored-procedure-and-permissions-is-execute-enough>

¹³<http://ezinearticles.com/?Avoiding-Publishing-and-Agent-Scams&id=197784>

¹⁴http://www.kyivpost.com/news/opinion/op_ed/detail/77654/print/

¹⁵<http://www.whoosh.org/issue28/schultz1.html>

¹⁶<http://www.psg.com/lists/radiusext/2007/msg00147.html>

¹⁷ $\sim\phi$ indicates the intension of ϕ .

¹⁸cf. Brasoveanu (2009) who proposes that the infelicity of superlative quantifiers in such cases is pragmatic, rather than semantic. For him, however, this does not follow from the meaning of these quantifiers, but is a stipulated felicity condition.

¹⁹<http://www.baltimorebeatdown.com/2010/10/8/1738491/ravens-pass-defense-vs-broncos-pass-offense>

²⁰<http://www.bluespace.com/blog/type1>

²¹www.cdhb.govt.nz/communications/healthline/issue61.pdf

²²<http://diamondgeezzer.blogspot.com/2004/04/youre-predictable-noon-deadline-has.html>

²³Though not, perhaps, for values of n that are *much* greater than 3.

²⁴As Bolinger (1978) points out, these negative questions actually have another reading, where PPIs are *not* licensed; under this reading, the speaker is seeking confirmation for the propositions that it is *not* cold, that the hearer is *not* tired, and that he does *not* have much money, respectively.

²⁵The other two are *scalar*, which is roughly the sense we have been dealing with so far in this paper, and *rhetorical retreat*, which is exemplified by:

(i)

a. Mary is at home—at least John's car is in the driveway.

b. Mary is at home—at least I think so.

c. Mary is at home—at least that's what Sue said.

d. Mary will help me—at least on the first draft, if it doesn't rain, when I've finished the outline,...

²⁶We are thankful to Barbara Partee for drawing our attention to this paper.

²⁷We make no claim here about the meaning of conditionals in general, but only about this specific construction.

²⁸http://www.google.de/url?sa=t&source=web&cd=1&ved=0CBsQFjAA&url=http://www.gamereport.com/poker/draw.html&ei=e-1fTYmFMZDGtAaM55C2CA&usg=AFQjCNG1VuDsckgNjH7mQLP-jXyuAn_cg

²⁹<http://www.google.de/url?sa=t&source=web&cd=11&ved=0CBsQFjAAOAO&url=http://faq.holdemmanager.com/glossary.php/letter/A&ei=G-5fTenqJ8n3sgalo422CA&usg=AFQjCNF4HPKGGKcf2Idzg0iHx7cBgSiq2nA>

³⁰Note that *think* is a bridge verb, allowing for wh-movement from its complement, and hence for wide-scope construal of scope-bearing elements.

³¹See Coniglio (2011) for such shifts of speaker-oriented expressions in German.

³²Here, and in the following section, we will only use intuitive formulations, leaving complete formalizations in terms of changes of commitment spaces for another occasion.

³³We leave open the question of which, exactly, these verbs are.

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